1. Ant odometer (150 million B.C.):

Saharan desert ants may have built in "pedometers" that count steps and allow the ants to measure exact distances. Ants with stilts glue to their legs travel too far and pass their nest entrance, suggesting that sterile length is important for distance determination.

2. Primate count (30 Million B.C.)

Around 60 million years ago, small, lemur-like primates had evolved 30 millions areas of the world, and 30 million years ago, primates with monkeylike characteristics existed. Primates appear to have some sense of numbers, and the higher primates can be taught to identify numbers from 1 to 6 by processing the appropriate computer key when shown a certain number of objects.

3. Cicada generated primes numbers (1 million B.C.)

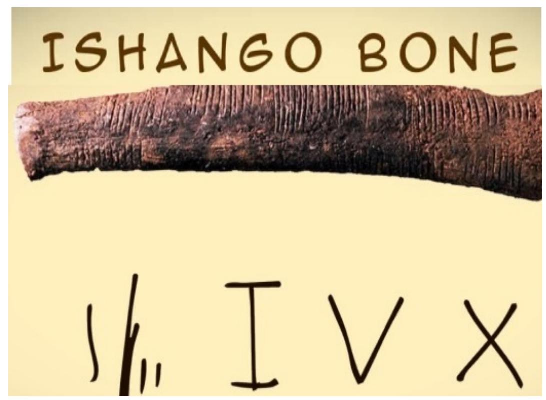
Cicadas are winged insects that evolved around 1.8 million years ago during the Pleistocene epoch. Cicadas of the genus *Magicicada* spend most of their lives below the ground, feeding on the juices of plant roots, and then emerge, mate, and die quickly. These creatures display a startling behavior: Their emergence is synchronized with periods of years that are usually the prime numbers 13 and 17. This research is still in its infancy and many questions remain. What is special about 13 and 17? What predators or parasites have actually existed to drive the cicadas to these periods?

4. **Knots (100,000 B.C.)**

The quintessence of ornamental knots is exemplified in the book of Kells, an ornately illustrated Gospel Bible, produced by Celtic monks in about A.D.800. In modern times, the study of knots, such as the trefoil knot with the 3 crossings, is part of a vast branch of mathematics dealing with closed twisted loops. In 1914, German mathematician Max Delm (1878-1952) showed that the trefoil knots mirror images are not equivalent. For centuries, mathematicians have tried to develop ways to distinguish tangles that look like knots (called *unknots)* from true knots and to distinguish true knots from one another. Today, knot theory in mathematics has become so advanced that mere mortals find it challenging to understand its most profound applications.

5. **Ishango Bone (18,000 B.C.)**

The Ishango baboon bone (found in Ishango, near the headwaters of the Nile River), with its sequence of notches, was first thought to be a simple tally stick used by a Stone Age African. These bones suggest a simple understanding of doubling or halving. However, some scientist believes that the marks suggest a mathematical prowess that goes beyond counting of objects. The full mystery of Ishango bone can't be solved until other similar bones are discovered.



6. **Quipu (3000 B.C.)**

The ancient Incas of South America used quipus (pronounced "key-poos"), memory banks made of strings and knots for storing numbers made of knotted strings to store numbers. Knots types and positions, cord direction, cord level and colors often represented dates and counts of people and objects. The quipus may have contained more information such as construction plans, dance patterns, and even aspects of Inca history. The quipu is significant because it dispels the notion that mathematics flourishes only after a civilization has developed writing.

7. Dice (3000 B.C.)

Dice was originally made from the anklebones of animals and the oldest known dice was found in the southeastern Iran. Dice was among the earliest means for producing random numbers. And now a days it used for finding probability also. In ancient civilizations, people used dice to predict the future, believing that the Gods influenced dice outcomes.

8. Magic square (2200 B.C.)

Magic squares originated in China and were first mentioned in a manuscript from the time of Emperor Yu. A magic square consists of N^2 boxes, called Cells, filled with integers that are all different. The sum of the numbers in the horizontal rows, vertical columns and diagonals are equal. Indian mathematician Shrinivas Ramanujan's magic square is a well known example.

9. Plimpton 322 (1800 B.C.)

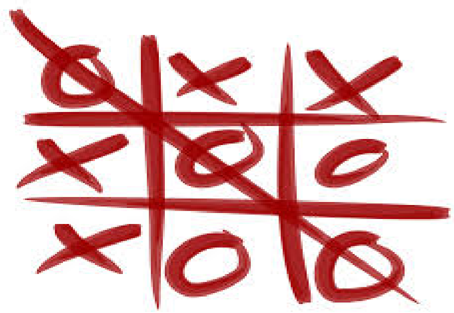
Plimpton 322(It named after a New York publisher George Plimpton who, in 1922, bought the tablet for $10 from a dealer and then donated it to a Columbian University) refers to a mysterious Babylonian clay tablet featuring numbers in a cuneiform script in a table of 4 columns and 15 rows. The tables specify the Pythagorean triples- i.e., whole numbers that specify the Pythagorean Theorem and the 4th column in the table simply contain the row number.

10. Rhind Papyrus (1650 B.C.)

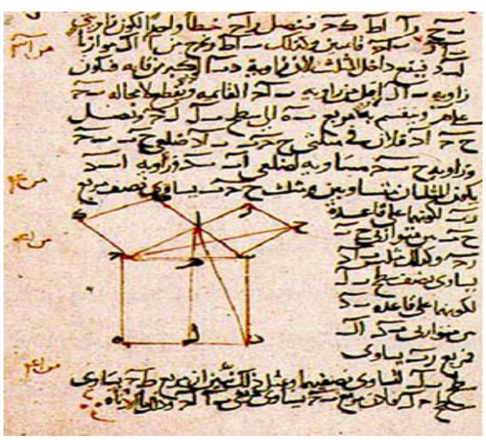
The rhind papyrus is the most important source of information concerning ancient Egyptian mathematics. This scroll, about a foot of 30 cm high and 18 feet (5.5 m) long was found in a tomb in Thebes on the east bank of the river Nile. It includes mathematical problems involving fractions, A.P., algebra, geometry and accounting.

11. Tic Tac Toe (TTT) (1300 B.C.)

T.T.T. is among humanity's best known & most ancient games. This game was first played in Egypt. For TTT,2 players 0 & X, take turn making their symbols in the spaces of a 3x3 grid. The player who first places 3 of his own marks in a horizontal, vertical or diagonal row wins.

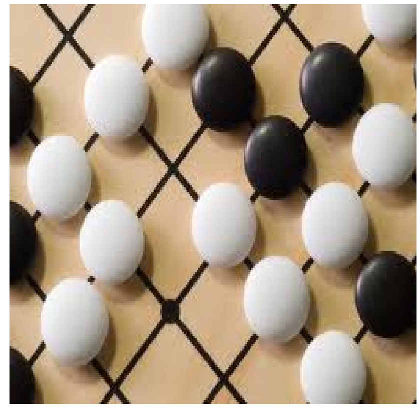


12. Pythagorean triangle & theorem (800 B.C)



Pythagorean triangles were probably known even earlier as "Babylonians". Although Pythagoras is often credited with the formulation of the Pythagorean Theorem in about 580 B.C. but evidence suggest that theorem was developed by Hindu mathematician Baudhayan in about 800B.C. in his book *Baudhayana sulba sutra*. It states that the square of hypotenuse length is equal to the sum of the square of other two lengths in a right angle triangle.

13.Go Game (548 B.C.)



Go is a 2 player's game, the 2 players that originated in China around 2000 B.C., alternatively place black & white stones on the intersections of a 19x19 playing board. A stone or a group of stones is captured & removed if it is tightly surrounded by the stones of opposite color. The objective is to control a larger territory that's one opponent. Go is complex, due in part to the larger game board, complicated strategies & huge numbers of variations in possible game.

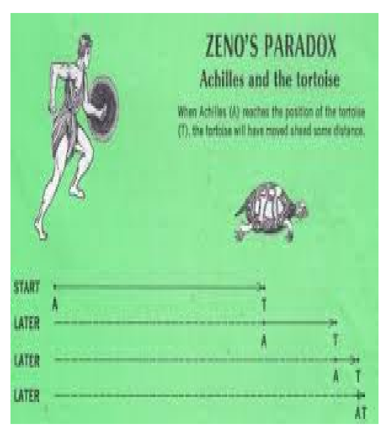
Chess is a smaller version of Go Game

14.Mathematical brotherhood [Pythagoras] (530 BC)

Pythagoras moved to Croton, Italy, to teach mathematics, music & reincarnation. He observed that vibrating strings produce harmonious sounds when the ratios of the length of the strings are whole numbers. He also studied triangular numbers (based on the patterns of the dots in a triangular shape) and perfect numbers (integers that are the sum of their proper positive divisors)

15.Zeno's Paradoxes (445 B.C.)

Greek philosopher Zeno invented a set of four paradoxes dealing with counterintuitive aspects of continuous space and time.

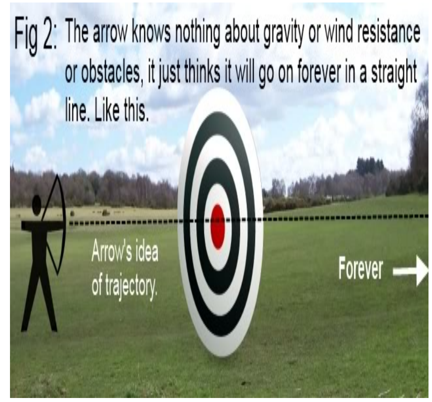


1. Dichotomy paradox: Before an object can travel a given distance d, it must travel a distance d/2. In order to travel d/2, it must travel d/4, etc. Since this sequence goes on forever, it therefore appears that the distance d cannot be traveled. The resolution of the paradox awaited calculus and the proof that infinite geometric series such as sum_(i=1)^(infinity)(1/2)^i=1 can converge, so that the infinite number of "half-steps" needed is balanced by the increasingly short amount of time needed to traverse the distances.

2. Achilles and the tortoise paradox: A fleet-of-foot Achilles is unable to catch a plodding tortoise which has been given a head start, since during the time it takes Achilles to catch up to a given position, the tortoise has moved forward some distance. But this is obviously fallacious since Achilles will clearly pass the tortoise! The resolution is similar to that of the dichotomy paradox.

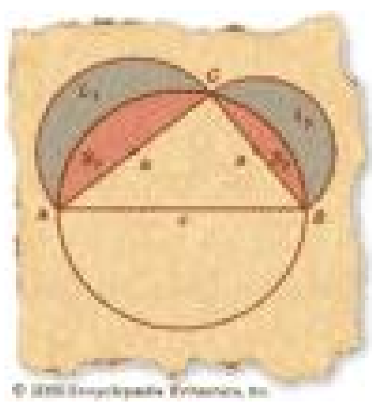
i.e.1=1/2+1/4+1/8+1/16+1/32+⋯

3. **Arrow paradox**: An arrow in flight has an instantaneous position at a given instant of time. At that instant, however, it is indistinguishable from a motion - less arrow in the same position, so how is the motion of the arrow perceived?



4**. Stade paradox**: A paradox arising from the assumption that space and time can be divided only by a definite amount.

16. Quadrature of lune(460 B.C.)

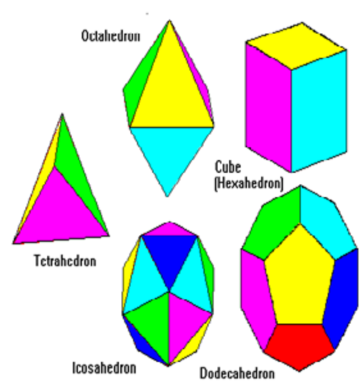


Hippocrates of Chaos demonstrated that the moon-shaped areas between circular arcs, known as lunes, could be expressed exactly as a rectilinear area, or quadrature.

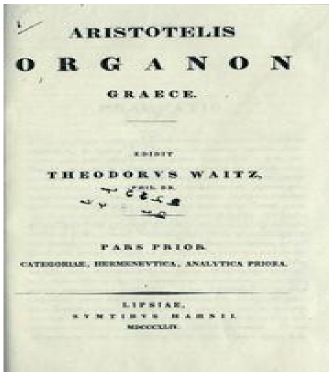
Hippocrates managed to square several sorts of lunes, some on lunes, some on arcs greater and less than semicircles and he intimated, though he may not have believed, that his method could square an entire circle. At the end of the classical age, **Boethius** (c. AD 470–524), who Latin translations of snippets of Euclid mentioned that someone had accomplished the squaring of the circle. Whether the unknown genius used lunes or some other method is not known, since for lack of space Boethius did not give the demonstration. He thus transmitted the challenge of the quadrature of the circle together with fragments of geometry apparently useful in performing it. Europeans kept at the hapless task well into the Enlightenment. Finally, in 1775, the Paris Academy of Sciences, fed up with the task of spotting the fallacies in the many solutions submitted to it, refused to have anything further to do with circle square's.

17. Platonic solids (350 B.C.)

Plato described the five Platonic solids in *Timaeus* in around 350 B.C. A *Platonic solid is* a convex multifaceted 3-D object whose faces are all identical polygons, with sides of equal length and angles of equal degrees. A Platonic solid also has the same number of faces meeting at every vertex. The best-known example of a Platonic solid is the cube, whose faces are six identical squares. The ancient Greeks recognized and proved that only five Platonic solids can be constructed: the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. For example, the icosahedron has 20 faces, all in the shape of equilateral triangles



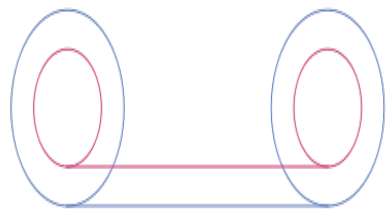
18. Aristotle’s organons (350B.C.)



Aristotle wrote six works that were later grouped together as the *Organon*, which means “instrument.” These works are the *Prior Analytics, Posterior Analytics, On Interpretation, Topics, Sophistical Refutations,* and *Categories.* These texts are considered the body of Aristotle’s work on logic, though there is a great deal in the *Organon* that we would not consider logic, and many of Aristotle’s other works, most notably the *Metaphysics,* deal to some extent with logic. These six works have a common interest not primarily in saying what is true but in investigating the structure of truth and the structure of the things that we can say such that they can be true. Broadly speaking, the *Organon* provides a series of guidelines on how to make sense of things.

19. Aristotle's wheel paradox (350 B.C.)

Aristotle's wheel paradox is a paradox from the Greek work Mechanica traditionally attributed to Aristotle. There are two wheels, one within the other, whose rims take the shape of two circles with different diameters. The wheels roll without slipping for a full revolution. The paths traced by the bottoms of the wheels are straight lines, which are apparently the wheels' circumferences. But the two lines have the same length, so the wheels must have the same circumference, contradicting the assumption that they have different sizes: a paradox.



The fallacy is the assumption that the smaller wheel indeed traces out its circumference, without ensuring that it, too, rolls without slipping on a fixed surface. In fact, it is impossible for both wheels to perform such motion. Physically, if two joined concentric wheels with different radii were rolled along parallel lines then at least one would slip; if a system of cogs were used to prevent slippage then the wheels would jam. A modern approximation of such an experiment is often performed by car drivers who park too close to a curb. The car's outer tire rolls without slipping on the road surface while the inner hubcap both rolls and slips across the curb; the slipping is evidenced by a screeching noise.

Alternatively, the fallacy is the assumption that the smaller wheel is independent of the larger wheel. Imagine a tire as the larger wheel, and imagine the smaller wheel as the interior circumference of the tire and not as the rim. The movement of the inner circle is dependent on the larger circle. Thus its movement from any point to another can be calculated by using an inverse of their ratio.

20. Euclid's elements (300 B.C.)

Greek mathematician Euclid is known to almost every high school student as the author of The Elements, the long studied text on geometry and number theory. No other book except the Bible has been so widely translated and circulated. From the time it was written it was regarded as an extraordinary work and was studied by all mathematicians, even the greatest mathematician of antiquity -- Archimedes, and so it has been through the 23* centuries in many languages starting, in the original Greek, then in Arabic, Latin, and many modern languages that have followed

Postulates :

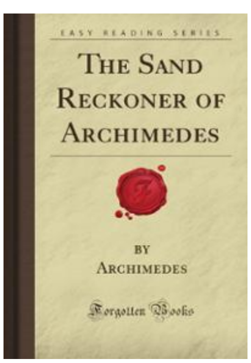
1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.
4. Those all right angles are equal to one another.

Axioms:

1. That, if a straight line falling on two straight lines make the interior angles on the same side less than to right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater that the part.

Contradiction-* 23 century or 24 century

21. Archimedes: Sand reckoner (250 B.C.)

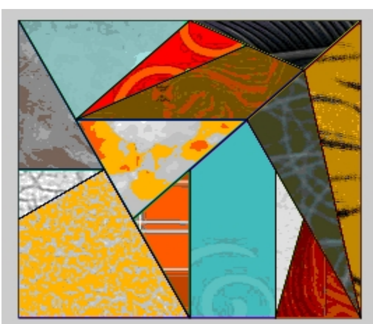


Syracuse, Italy's mathematician Archimedes, the Sand Reckoner is a remarkable work in which Archimedes proposes a number system that uses powers of a myriad *myriad* (base 100,000,000) and is capable of expressing numbers up to 8 x 1063 in modern notation.

22. Archimedes: Cattle problem (250 B.C.)

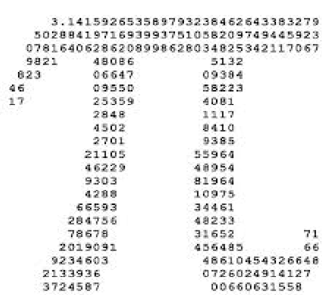
In 1769, Gotthold Ephraim Lessing was appointed librarian of the Herzog August Library in Wolfenbüttel, Germany, which contained many Greek and Latin manuscripts. A few years later, Lessing published translations of some of the manuscripts with commentaries. Among them was a Greek poem of forty-four lines, containing an arithmetical problem which asks the reader to find the number of cattle in the herd of the *god of the sun*. The name of Archimedes appears in the title of the poem, it being said that he sent it in a letter to Eratosthenes to be investigated by the mathematicians of Alexandria. The claim that Archimedes authored the poem is disputed, though, as no mention of the problem has been found in the writings of the Greek mathematicians. The general solution was found in 1880 by A. Amthor. He gave the exact solution using exponentials and showed that it was about 7.76×10^{206544} cattle, far more than could fit in the observable universe. The decimal form is too long for humans to calculate exactly, but multiple precision arithmetic packages on computers can easily write it out explicitly.

23. Archimedes: Stomachian puzzle (250 B.C.)



The Stomachion is a puzzle that is at least 2,200 years old. It was known to the ancient Greeks. Some people think that it was created by the Greek scientist Archimedes, which is why it is sometimes called Archimedes' Puzzle or the *Loculus* of Archimedes. The puzzle consists of 14 pieces of various shapes and sizes. These pieces are created by dividing a square as shown below. The object of the puzzle is to rearrange the pieces to form other shapes.

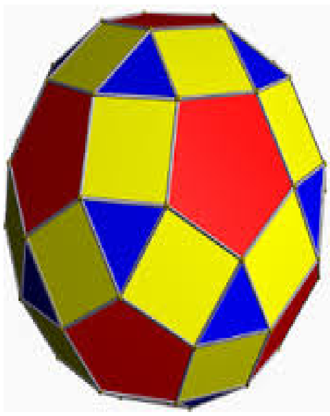
24. π (250 BC)



Pi, symbolized by the Greek letter π, is the "ratio of a circle's circumference to its diameter and is approximately equal to 3.14159. Perhaps ancient peoples observed that for every revolution of a cartwheel, a cart moves forward about three times the diameter of the wheel-an early recognition that the circumference is about three times the diameter. An ancient Babylonian tablet states that the ratio of the circumference of a circle to the perimeter of an inscribed hexagon is I to 0.96, implying a value of pi of 3.125.

Greek mathematician Archimedes (c. 250 B.C.) was the first to give us a mathematically rigorous range for ""-a value between 223/71 and *22/7.* The Welsh mathematician William Jones (1675-1749) introduced the symbol π in 1706, most likely after the Greek word for periphery, which start, with the letter "π".

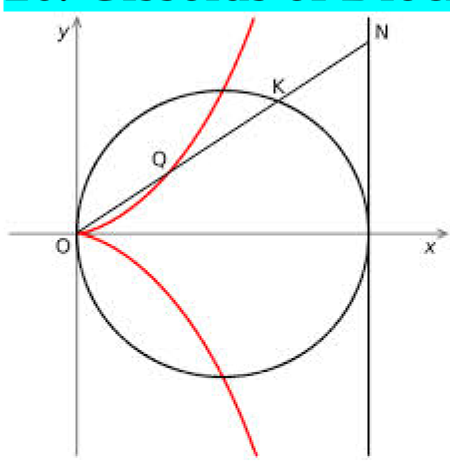
25. Archimedean Semi-Regular Polyhedra(240 BC)



Like Platonic Solids, Archimedean semi-regular polyhedra (ASRP) are convex, multifaceted 3-D objects whose faces are all regular polygons that have sides of equal length and angles of equal degrees. However, for the ASRP, the faces are of different kinds. For example, the polyhedron formed by IZ pentagons and 20 hexagons, which resembles a modem soccer ball, was described by Archimedes along with 12 other such polyhedra. Around every vertex (comer) of these kinds of solids, the same polygons appear **in** the same sequence-for example, hexagon-hexagon-triangle. Archimedes' original writings that described the 13 ASRP are lost and known only from other sources. During the Renaissance, artists. In 1619, Kepler presented the entire set in his book *Hannonices Mundi (The Hannonies of the World).*

ASRPs may be specified using a nume
For example, 3,5,3,5 means that a tri
order. Using this notation, we have the following ASRPs: 3,4.3,4 (a cuboctahedron); 3,5.3,5 (an icosidodecahedron); 3,6,6 (a truncated tetrahedron); 4,6,6 (a truncated octahedron); 3,8,8 (a truncated cube); 5,6,6 (a truncated icosahedron, or soccer ball); 3,10,10 (a truncated dodecahedron); 3,4,4,4 (a rhombicuboctahedron); 4,6,8 (a truncated cuboctahedron); 3,4,5,4 (a rhombicosidodecahedron); 4,6,10 (a truncated icosidodecahedron); 3,3,3,3,4 (a snub cube, or snub cuboctahedron); and 3,3,3,3,5 (a snub dodecahedron, or snub icosidodecahedron).
The 32-faced truncated icosahedron is particularly fascinating. Soccer ball shapes are based on this Archimedean solid.

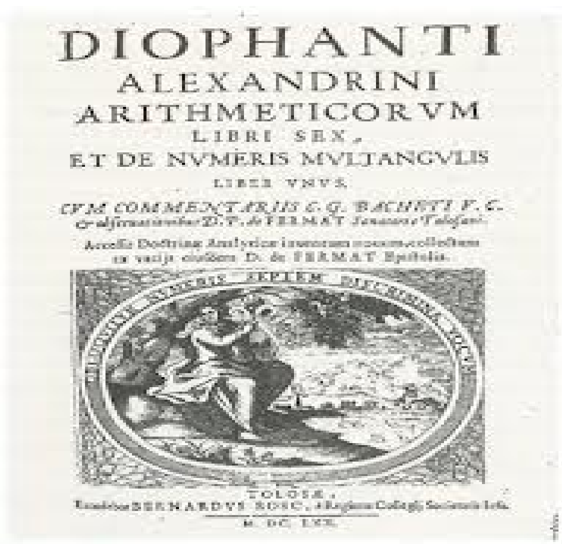
26. Cissoids of Diocles(180 BC)



The cissoids of Diocles was discovered by Greek mathematician Diocles, around 180 B.C., during his attempts to use its remarkable properties to double a cube. "Doubling the cube" refers to a famous and ancient challenge of constructing a cube with a volume twice the volume of a given smaller cube, which means that the larger cube has an edge that is *3* times larger than the first cube. Diocles' use of the cissoid, and its intersection with a straight line, was theoretically correct, but did not rigorously follow the rules of Euclidean construction that allowed the use of only a compass and a straightedge.

The name *cissoid* comes from the Greek term meaning "ivy-shaped." The graph of the curve extends to infinity along both directions of the y-axis and has a single cusp. Both branches of the curve that extend away from the cusp approach the same vertical asymptote. If we draw a circle that passes through the cusp at point 0 and that is tangent to the asymptote, then *any* line joining the cusp and a point M on the cissoid can be extended so that it intersects the asymptote at B. The length of the linear extension from C to B *is* always equal to the length between 0 and M. The curve may be expressed in polar coordinates as $r = 2a(\sec\theta - \cos\theta)$ or in rectangular coordinates as $r^2 = x^3/(2a - x)$. Interestingly, the Cissoid can be produced by tracing the vertex of a parabola as it rolls, without slipping, on a second parabola of the same size. Diocles was fascinated by curves known as conic *sections,* and in his work On *Burning* Mirrors he discussed the focal point of a parabola. **One of his goals was to find a mirror surface that focuses the maximum amount of heat when it is placed in sunlight.**

27. Diophantus's *Arithmetica(250 BC)*



Greek mathematician Diophantus of Alexandria, sometimes called the "futher of algebra;' was the author of *Arithmetica* (c. 250), a series of mathematical texts that has influenced mathematics for centuries. *Arithmetica,* the most fumous work on algebra in all of Greek mathematics, contains various problems along with numerical solutions to equations. Diophantus is also important due to his advances in mathematical notation and his treatment of fractions as numbers. In the dedication to *Arithmetica,* Diophantus writes to Dionysus that although the material in the book may be difficult, "it will be easy to grasp, with your enthusiasm and my teaching."

Diophantus's various works were preserved by the Arabs and translated into Latin in the sixteenth century. Diophantine equations, with their integer solutions, are named in his honor. Pierre de Fermat scribbled his famous Fermat's Last Theorem involving integer solutions of $a^n + b^n = c^n$ in a French translation of *Arithmetica,* published in 1681. Although the Babylonians were aware of some methods for solving linear and quadratic equations of the kind that fascinated Diophantus.
The rediscovery of *Arithmetica* through Byzantine sources greatly aided the renaissance of mathematics in Western Europe and stimulated many mathematicians, of whom the greatest was Fermat.

28. Discovery of zero(650 AD)

Though undoubtedly taken for granted today, the number (or lack thereof) known as "**zero**" was not always a part of the human mathematical mindset. Since zero is more of a concept than an actual number, the development of 'true zero' took quite some time to enter into human consciousness.

Zero was known is Indians well before others an ancient Indian astronomer named ARYABHATTA has discovered zero for his complex astronomical calculation.
The first recorded zero is attributed to the Babylonians in the 3rd century B.C. – A long period followed when no one else used a zero place holder. But then the Mayans, halfway around the world in Central America, independently invented zero in 4th Century CE. The final independent invention of zero in India was long debated by scholars but seems to be set around the Middle of 5th century. It spread to Cambodia around the end of 7th century.
From India it moved to China and then to the Islamic countries zero finally reached Western countries (Europe) in 12th century.

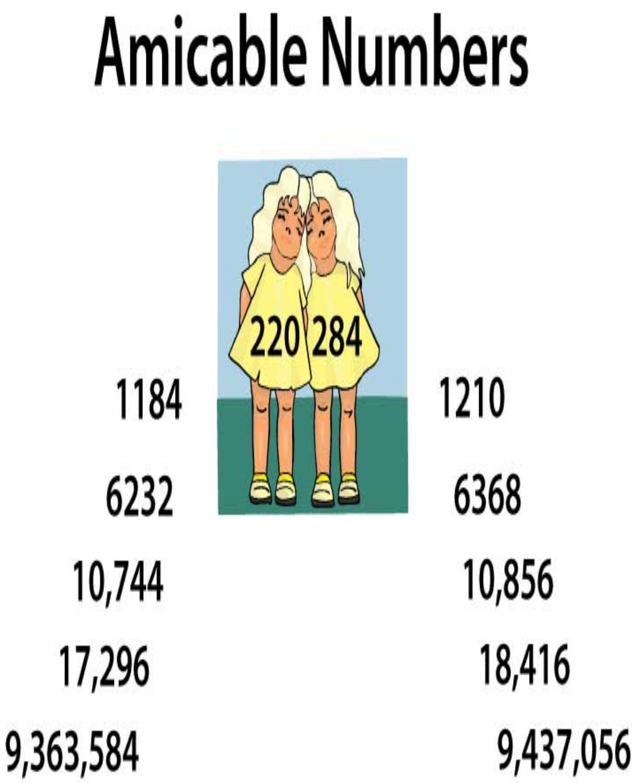
29. Al-Khwarizmi's algebra(830 AD)

Abu Jafar Muhammed Ibn e Musa Al Khwarizmi , an Iranian mathematician, also known as the Father of algebra (Dtiophantus also called father of algebra). He was a scholar at the academy "House of wisdom" where he publishing multiple treatise during the rule of Al ma'mun (813-833). The most famous of these treatises is Hisab Al Jabr W'al-muqabala (treatise on algebra). He provides a very practical take on algebra in the text. He begins by defining natural numbers in an original useful way. From there, he proposes how to solve linear & quadratic equations. All of his work any typical algebraic notation seen today; instead the example problem he provides are all written out in Arabic.

For al-Khwarizmi, al-jabr is a method in which we can eliminate negative quantities in an equation by adding the same quantity to each side. For example, we can reduce $X^2=50X-5X^2$ to $6X^2-50X$ by adding $5X^2$ to both sides. *Al-muqabala* is a method Whereby we gather quantities of the same type to the same side of the equation.
For Example $X^2+15=X+5$ is reduced to $X^2+10=X$

The book helped readers to solve equations such as those of the forms $X^2 +10X=39$, $X^2+21=10X$ and $3X+4=X^2$, but more generally, al-Khwarizmi believed that the difficult mathematical problems could be solved if broken down into a series of Smaller steps. Al-Khwarizmi intended his book to be practical, helping people to make
Calculations that deal with money, property inheritance, lawsuits, trade, and the digging Of canals. His book also contained example problems and solutions

30. Thabit Amicable Number(850 AD)



Amicable numbers are two different numbers so related that the sum of the proper divisor of each of number is equal to the other number.
Example:- (220,284)
The proper divisor of 220 is 1,2,4,5,10,11,20,22,44,55,110 and the sum of these numbers is 284.
The proper divisor of 284 is 1, 2,4,71 & 144 and the sum of these numbers is 220.
More Amicable numbers are (1184,1210),(2620,2924),(5020,5564),(6362,6368).
These numbers were known to the Pythagoreans, who credited them with many mystical properties .A general formula by which some of these numbers could be derived was circa 850 by Iraqi mathematician Thabit-ibn-Qurra (826-901). Other Arab mathematicians who studied amicable the mathematical numbers are Al-Majriti , Al-Baghdadi and Al-farsi. The Iranian mathematician Mohd. Baqir Yazdi (16th century) discovered the pair (9363584, 9437056) though this has often been attributed to Desecrates.

Thabit theorem is a method for discovering Amicable numbers invented in 9th century .It states that $P=3 \times 2^{n-1} -1$

$Q=3\times 2^n -1$

$R=9\times 2^{2n-1}-1$, where $n>1$, is a integer & P, Q, R are prime numbers.

Then $2^n \times P\times Q$ and $2^n\times R$ is pair of amicable number.

This formula gave only three amicable numbers. That are (220,284), (17296, 18416), (9363584, 9437056).

Euler rule also gave a rule for amicable number i.e.

$P= [(2^{n-m}+1) \times 2^m-1]$

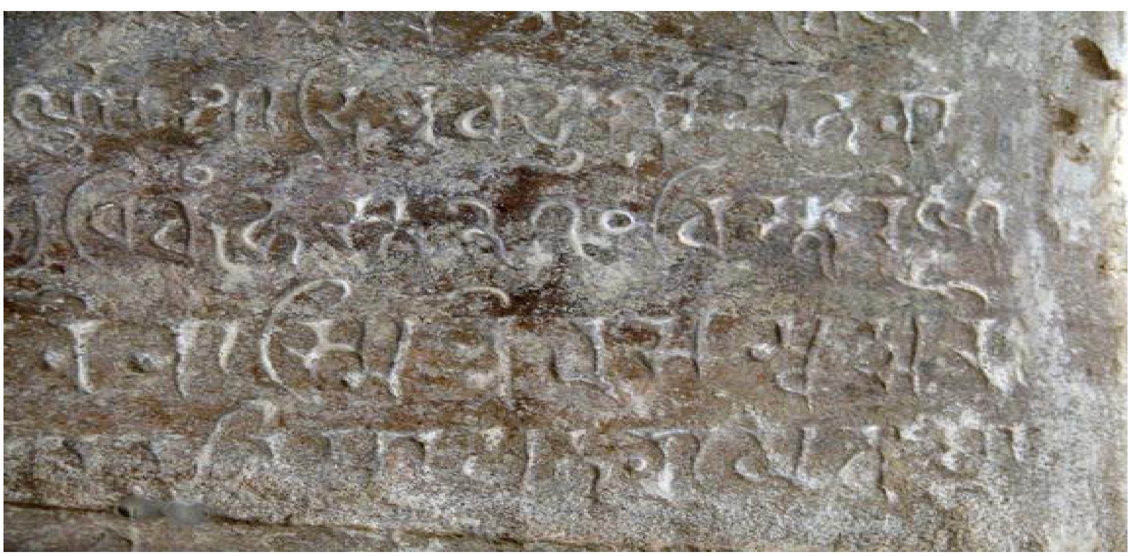
$q = [[(2^{n-m}+1) \times 2^n-1]$

$r= [[(2^{n-m}+1)^2\times 2^{m+n}-1]$ where $n>m>0$ are integer & p, q, r are prime no.

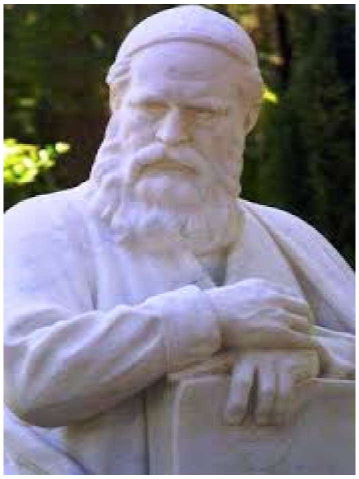
Then $2^n \times p \times q$ & $2^n \times r$ are a pair of available no

31. Chapters in Indian Mathematics(953 AD)

Al-Uqlidisi ("the Euclidian") was an Arab mathematician whose *Kitab al-fusul Ii al-hioob ai-Hindi (Chapters in Indian Mathematics) is* the earliest-known Arabic work discussing the positional use of the Hindu-Arabic numerals, meaning the use of digits corresponding to 0 through 9 in which each position starting from the right of a multi-digit number corresponds to a power of 10 (for example, I, 10, 100, and 1,000). AI-Uqlidisi's work also represents the earliest-known arithmetic extant in Arabic. Although al-Uqlidisi was born and died in Damascus, he was well traveled and may have learned about Hindu mathematics in India. Only one copy of this manuscript remains today. Al-Uqlidisi also discussed the problems of previous mathematicians in terms of the new system of numerals. Dick Teresi, the author of several books about science and technology, writes, "His name was evidence of his reverence for the Greeks. He copied the works of Euclid, hence the name al-Uqlidisi. One of his legacies is paper-and-pen mathematics." During al-Uqlidisi's time, it was common in India and the Islamic world to perform mathematical calculations in the sand or in dust, erasing steps with one's hand as one proceeded. Al-Uqlidisi suggested that paper and pen be used instead. Written arithmetic preserves the process, and although his scheme did not involve erasure of ink numbers, it did permit greater flexibility in calculation. In a sense, paper drove the evolution of modem methods for performing multiplication and long division. Regis Morelon, the editor of the *Encyclopedia* of *the* History of *Arabic Science,* writes, "One of the most remarkable ideas in the arithmetic of al-Uqlidisi *is* the use of decimal fractions" and the use of the decimal symbol. For example, to halve 19 successively, al-Uqlidisi gave the following: 19,9.5,4.75,2.375,1.1875,0.59375. Eventually, the advanced calculations enabled by the decimal system led to its common use throughout the region and the world.



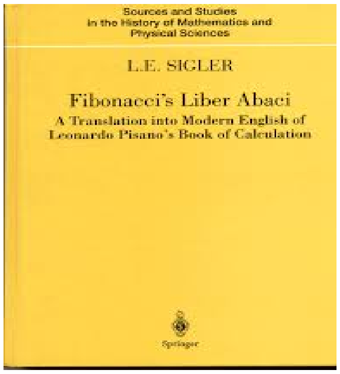
32. Omar Khayyam treatise(1070 AD)



Omar Khayyam is the Persian mathematician, astronomer, philosopher and poet. He is best known for his collection of poems, *the rubiyyat of omar Khayyam* **.** However he also written a treatise on *Demonstration of problem of Algebra (1070)*. **I**n this treatise, he derived method for solving some cubic and higher order equations. An example of cubic equation that he solved $x^3 + 200x = 200x + 2000$. His treatise contains a comprehensive classification of cubic equations with geometric solutions found by means of intersecting conic sections.

Khayyam also show how to obtain the nth power of the binomial a+b in powers of a & b, when n is the whole number. Khayyam 1077's work on geometry, *sharh ma ashkala min musadarat kitab uqlidis (commentaries on the difficulties in the postulate's of Euclid book),* provides an interesting look at Euclid's famous parallel postulates. In sharh, he discussed properties of Non Euclidean Geometries and thus stumbled into a realm of mathematics that would not flourish until the 1800.

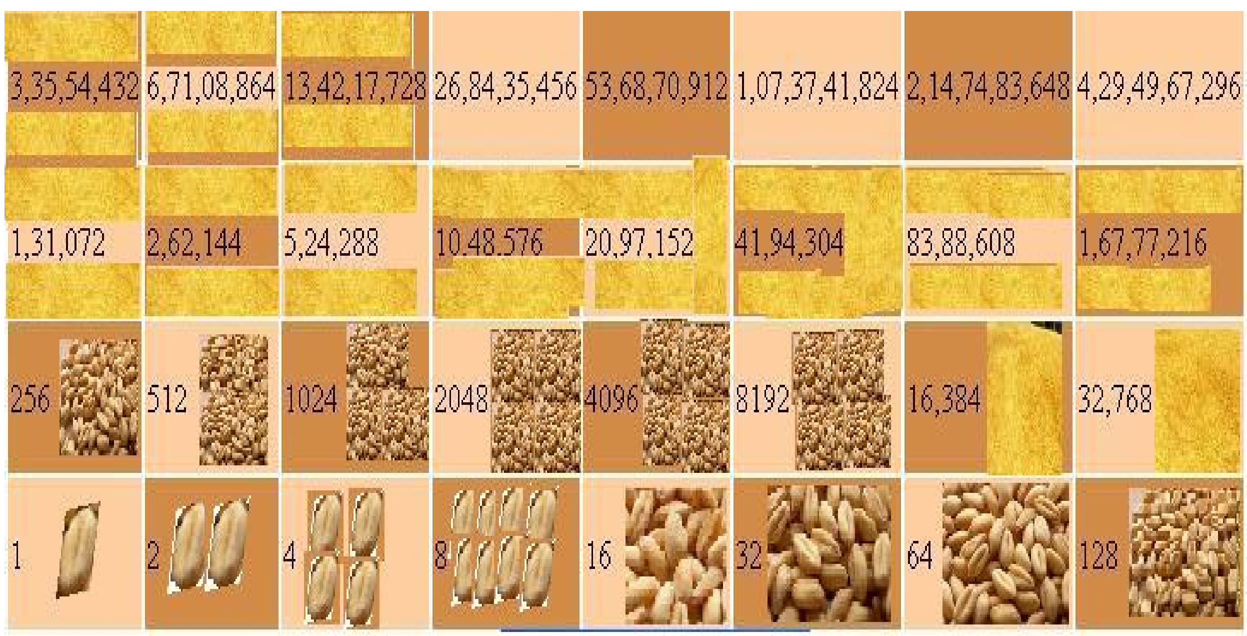
33. Fibonacci's Liber Abaci(1202 AD)



Carl Boyer refers to the Leonardo of Pisa also known as Fibonacci, a wealthy Italian merchant, traveled through Egypt, Syria and Barbary (Algeria) and in 1202 published a book Liber Abaci (The book of Abacus). This introduced the world about Hindu-Arabic numeral system and decimal number system. Fibonacci notes, "These are the nine figures of Indians: 9,8,7,6,5,4,3,2,1. With these nine figures, and with the sign 0, which in Arabic is called Zepirum, any number can be represented as well as be demonstrated". This book also introduced Western Europe to the famous number sequence 1,1,2,3,5,8,13,… which today is called the Fibonacci sequence. These numbers appear in an amazing number of mathematical disciplines and in nature.

The arrangement of seeds in a sunflower can be understood using the Fibonacci numbers. Sunflower heads, like those of other flower, contain families of interlaced spiral of seeds- one spiral is winding clockwise and the other is counterclockwise. The number of spiral in each heads as well as the number of petals in the flower is the Fibonacci number.

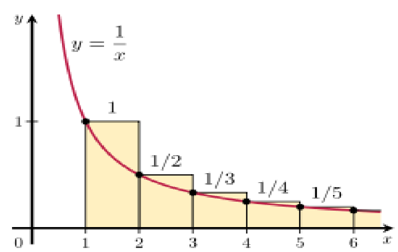
34. Wheat on chessboard(1256 AD)



About 1260 AD*, Ibn Khallikan, a Kurdish historian living in the Abbasid Empire (modern Iraq), wrote an encyclopedia with biographies of many famous men (though no women). One of the biographies includes a story about chess and the meaning of "exponential growth." The story takes place in India, because Ibn Khallikan knew that chess was a game that came from India. According to this story, King Shihram was a tyrant who oppressed his subjects. One of his subjects, a wise man named Sissa ibn Dahir, invented the game of chess for the king to play, to show him that a king needed all his subjects and should take good care of them. King Shihram was so pleased that he ordered that the game of chess should be preserved in the temples, and said that it was the best thing he knew of to train generals in the art of war, a glory to religion and the world, and the foundation of all justice. Then King Shihram asked Sissa ben Dahir what reward he wanted. Sissa answered that he didn't want any reward, but the king insisted. Finally Sissa said that he would take this reward: the king should put one grain of wheat on the first square of a chessboard, two grains of wheat on the second square, four grains on the third square, eight grains on the fourth square, and so on, doubling the number of grains of wheat with each square (an exponential rate of growth). The solution is to compute the sum of first 64 term of geometrical progression, i.e. 1+2+2^2+…+2^63=2^64-1, which is a walloping 18,446,744,073, 709,551,615 grains of wheat. He ordered his slaves to bring out the chessboard and they started putting on the wheat. Everything went well for a while, but the king was surprised to see that by the time they got halfway through the chess board the 32nd square required more than four billion grains of wheat, or about 100,000 kilos of wheat. Now Sissa didn't seem so stupid anymore. Even so, King Shihram was willing to pay up. But as the slaves began on the second half of the chessboard, King Shihram gradually realized that he couldn't pay that much wheat - in fact, to finish the chessboard you would need as much wheat as six times the weight of all the living things on Earth.

*contradiction…1260 AD or 1256

35. Harmonic series diverges(1350 AD)

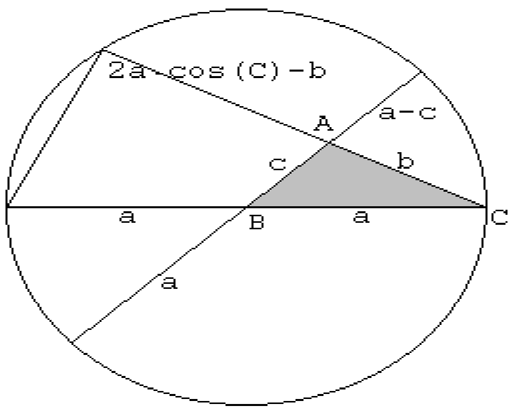


If God were infinity, then *divergent series* would be His angels flying higher and higher to reach Him. Given an eternity, all such angels approach their Creator. For example, consider the following infinite series: I + 2 + 3 + 4 ... . If we add one term of the series each year, in four years the sum will be 10. Eventually, after an infinite number of *years,* the sum reaches infinity. Mathematicians call such series divergent because they explode to infinity, given an infinite number of terms.

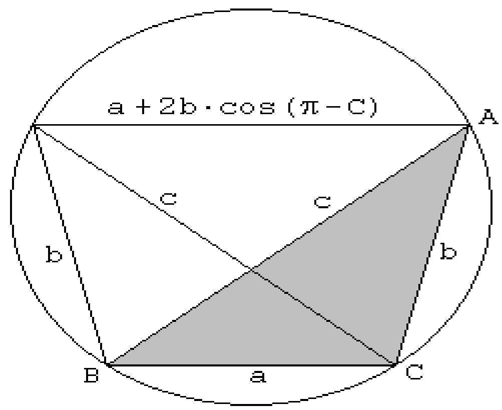
Nicole Oresme, the famous French philosopher of the Middle Ages, was the first to prove the divergence of the harmonic series (c. 1350). His results were lost for several centuries, and the result was proved agam by Italian mathematician Pietro Mengoh in 1647 and by Swiss mathematician Johann Bernoulli in 1687. His brother Jacob Bernoulli published a proof in his 1689 work *Tractatus* de *Seriebus Infinitis (Treatise* on *Infinite Series),* which he closes with: "So the soul of immensity dwells in minutia. And in narrowest limits no limits inhere. What joy to discern the minute in infinity! The vast to perceive in the small, what divinity!"

36. Law of Cosine(1427 AD)

Euclid's Element contains the seeds of concepts that lead to the law of cosine. In the Fifteenth century, the Persian astronomer and mathematician Ghiyath al Din Jamshid Mas'ud al Kashi provide accurate trigonometric tables and expressed the theorem in a form suitable for modern usage. French mathematician Francois Viete discovered the law independently of al Kashi. In France, the law of cosine is known as the Theoreme d'al Kashi, after al Kashi's unification of existing work on the subject. Al Kashi's most important work is The Key of Arithmetic completed in 1427, which discusses the mathematics used in astronomy, surveying, architecture and accounting. He uses decimal fractions in calculating the surface area needed for certain muqarnas, decorative structure in Islamic and Persian architecture.



$$(2a\cdot\cos(C) - b)b = (a - c)(a + c)$$
$$c^2 = a^2 + b^2 - 2ab\cdot\cos(C)$$



$$c\cdot c = b\cdot b + (a + 2b\cdot\cos(\pi - c))\cdot a$$
$$c^2 = a^2 + b^2 - 2ab\cdot\cos(C)$$

The law of cosine may be used for calculating the length of one side of a triangle when the angle opposite this site and the length of other 2 sides are known. The law expressed as $c^2 = a^2 + b^2 – 2abCos(C)$, where a,b and c are the sides of a triangle and C is the angle between the sides a and b. because of its generality the application of law ranges from land surveying to calculating the flight path of aircraft.

37.Discoveries of series formula for π(1500 AD)

The series was independently discovered by many mathematicians at different places. German mathematician Gottfried Wilhelm Leibnitz in 1673, the Scottish mathematician and astronomer James Gregory in 1671 and the Indian mathematician in the 14^{th} or 15^{th} century whose identity is not definitely known, although the result is usually ascribed by Nilakantha Somyaji (1444-1544) in his book Tantrasangrah in 1500. Somyaji was aware that a finite series of rational numbers could never suffice to represent π.

Pi is symbolized by the Greek letter π, is the ratio of the circumference of the circle to its diameter and can be expressed by a formula π/4 = 1 - 1/3 + 1/5 - 1/7+….. And the arctan function(x)(this function is given by Gregory before Leibnitz) in trigonometry is also represented as x = x – x/3 + x/5 – x/7+…. Using the archtan series, the series for π/4 is obtained by setting x = 1.

The Discovery of the Series Formula for π by Leibniz, Gregory and Nilakantha

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1. Introduction

The formula for π mentioned in the title of this article is

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots. \quad (1)$$

One simple and well-known modern proof goes as follows:

$$\arctan x = \int_0^x \frac{1}{1+t^2}\,dt$$
$$= x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + (-1)^{n+1}\int_0^x \frac{t^{2n+2}}{1+t^2}\,dt.$$

The last integral tends to zero if $|x| \leq 1$, for

$$\left|\int_0^x \frac{t^{2n+2}}{1+t^2}\,dt\right| \leq \left|\int_0^x t^{2n+2}\,dt\right| = \frac{|x|^{2n+3}}{2n+3} \to 0 \quad \text{as } n \to \infty.$$

Thus, arctan x has an infinite series representation for $|x| \leq 1$:

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots. \quad (2)$$

The series for $\pi/4$ is obtained by setting $x = 1$ in (2). The series (2) was obtained independently by Gottfried Wilhelm Leibniz (1646–1716), James Gregory (1638–1675) and an Indian mathematician of the fourteenth century or probably the fifteenth century whose identity is not definitely known. Usually ascribed to Nilakantha, the Indian proof of (2) appears to date from the mid-fifteenth century and was a consequence of an effort to rectify the circle. The details of the circumstances and ideas leading to the discovery of the series by Leibniz and Gregory are known. It is interesting to go into these details for several reasons. The infinite series began to play a role in mathematics only in the second half of the seventeenth century. Prior to that, particular cases of the infinite geometric series were the only ones to be used. The arctan series was obtained by Leibniz and Gregory early in their study of infinite series and, in fact, before the methods and algorithms of calculus were fully developed. The history of the arctan series is, therefore, important because it reveals early ideas on series and their relationship with quadrature or the process of finding the area under a curve. In the case of Leibniz, it is possible to see how he used and

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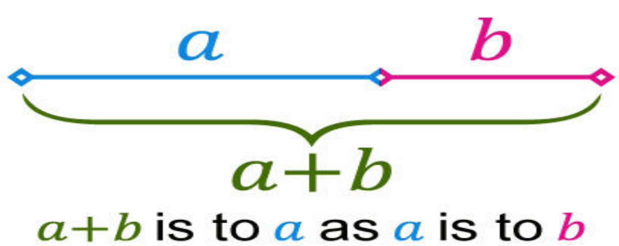
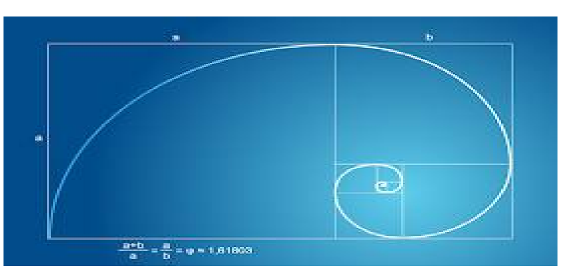
38. Golden ratio(1509 AD)



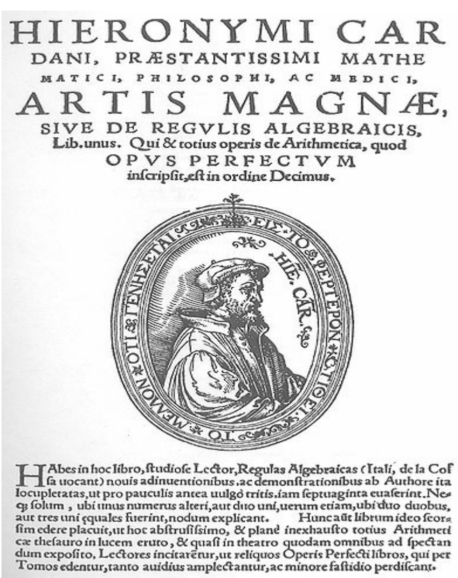
In 1509, a Italian mathematician Luca Pacioli, a close friend of Leonardo da vinci, published the Divina proportione, a treatise on a number that is now widely known as the Golden Ratio. We can easily understand the proportion by dividing a line into 2 segments so that the ratio of the whole segment to the longer parts is the same as the ratio of the longer part (b) to the shorter part (a) or (a +b)/b = b/a = 1.61803…



39. Loxodrome (1537 AD)

The loxodrome was invented by Portuguese mathematician and geographer Pedro Nunes. For the purposes of terrestrial navigation, the loxodromic spiral (also known as the spherical helix, loxodrome and rhumb line) goes through the north south meridians of the earth at a constant angle. The loxodrome coil is like a gigantic snake around the earth and spirals around the poles without reaching them. Loxodromic path allow the navigator to continually direct the vessel to the same point of the compass even though the path to the destination is longer.

40. Cardano's Ars Magna(1545 AD)



HIERONYMI CAR
DANI, PRÆSTANTISSIMI MATHE
MATICI, PHILOSOPHI, AC MEDICI,
ARTIS MAGNÆ,
SIVE DE REGVLIS ALGEBRAICIS,
Lib.unus. Qui & totius operis de Arithmetica, quod
OPVS PERFECTVM
inſcripſit,eſt in ordine Decimus.

HAbes in hoc libro,ſtudioſe Lector,Regulas Algebraicas (Itali, de la Coſſa uocant) nouis adinuentionibus,ac demonſtrationibus ab Authore ita locupletatas,ut pro pauculis antea uulgò tritis,iam ſeptuaginta euaſerint.Neq; ſolum , ubi unus numerus alteri,aut duo uni,uerum etiam,ubi duo duobus, aut tres uni æquales fuerint,nodum explicant. Hunc aũt librum ideo ſeorſim edere placuit,ut hoc abſtruſiſsimo, & planè inexhauſto totius Arithmeticæ theſauro in lucem eruto, & quaſi in theatro quodam omnibus ad ſpectandum expoſito, Lectores incitarētur,ut reliquos Operis Perfecti libros, qui per Tomos edentur,tanto auidius amplectantur,ac minore faſtidio perdiſcant.

The ***Ars Magna*** (Latin: "The Great Art") is an important book on Algebra written by Italian mathematician Girolamo Cardano. It was first published in 1545 under the title *Artis Magnæ, Sive de Regulis Algebraicis Liber Unus* (*Book number one about The Great Art, or The Rules of Algebra*). There was a second edition in Cardano's lifetime, published in 1570. It is considered one of the three greatest scientific treatises of the early Renaissance, together with Copernicus' *De revolutionibus orbium coelestium* and Vesalius' *De humani corporis fabrica*. The first editions of these three books were published within a two year span (1543–1545). The book, which is divided into forty chapters, contains the first published solution to cubic and quartic equations and negative numbers.

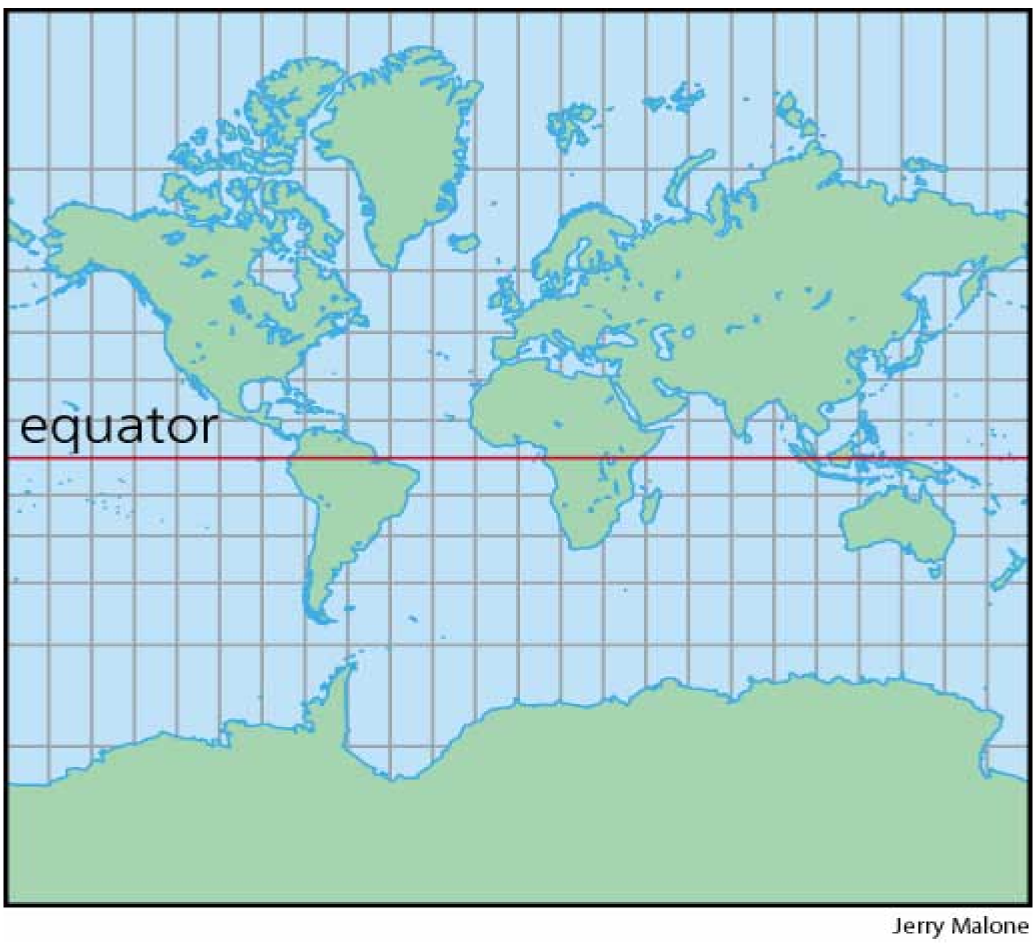
Cardano acknowledges that Tartaglia gave him the formula for solving a type of cubic equations and that the same formula had been discovered by Scipiano del Ferro. He also acknowledges that it was Ferrari who found a way of solving quartic equations. It is a common misconception that Cardano introduced complex numbers in solving cubic equations. Since (in modern notation) Cardano's formula for a root of the polynomial $x^3 + px + q$ is

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}},$$

square roots of negative numbers appear naturally in this context. However, $q^2/4 + p^3/27$ never happens to be negative in the specific cases in which Cardano applies the formula.

41. Mercator projection(1569 AD)

The Mercator projection is a cylindrical map projection presented by the Flemish geographer and cartographer Gerardus Mercator in 1569. It became the standard map projection for nautical purposes because of its ability to represent lines of constant course, known as rhumb lines or loxodromes, as straight segments which conserve the angles with the meridians. While the linear scale is equal in all directions around any point, thus preserving the angles and the shapes of small objects (which makes the projection conformal), the Mercator projection distorts the size and shape of large objects, as the scale increases from the Equator to the poles, where it becomes infinite



42. Imaginary numbers i (1572 AD)

In 1545, Girolamo Cardano wrote a book titled *Ars Magna.* He solved the equation x(10-x)=40, finding the answer to be 5 plus or minus√-15. Although he found that this was the answer, he greatly disliked imaginary numbers. He said that work with them would be, “as subtle as it would be useless”, and referred to working with them as “mental torture.”

Later, in 1637, Rene Descartes came up with the standard form for complex numbers, which is a+b*i*. He assumed that if they were involved, you couldn’t solve the problem. Lastly, he came up with the term “imaginary”, although he meant it to be negative. Issac Newton agreed with Descartes, and Albert Girad even went as far as to call these, “solutions impossible”. Although these people didn’t enjoy the thought of imaginary numbers, they couldn’t stop other mathematicians from believing that *i* might exist.

One of the ways people wanted to make them accepted was to be able to plot them of a graph. In this case, the X-axis is would be real numbers, and the Y-axis would be imaginary numbers. If the number were purely imaginary (like 2*i*), it would just be on the Y-axis. If the number was purely real, it would just be on the X-axis. The first person who considered this kind of graph was John Wallis.

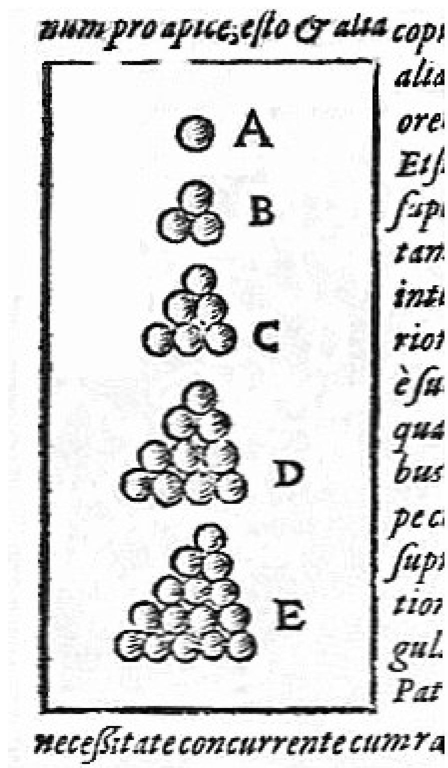
In 1685, he said that a complex number was just a point on a plane, but he was ignored. More than a century later, Caspar Wessel published a paper showing how to represent complex numbers in a plane, but was also ignored. In 1777, Euler made the symbol *i* stand for √-1, which made it a little easier to understand. In 1804, Abbe Buee thought about John Wallis’s idea about graphing imaginary numbers, and agreed with him. In 1806, Jean Robert Argand wrote how to plot them in a plane, and today the plane is called the Argand diagram. In 1831, Carl Friedrich Gauss made Argand’s idea popular, and introduced it to many people. In addition, Gauss took Descartes’ a+b*i* notation, and called this a complex number. It took all these people working together to get the world, for the most part, to accept complex numbers. In 1833, William Rowan Hamilton expressed complex numbers as pairs of real numbers (such as 4+3*i* being expresses as (4,3)), making them less confusing and even more believable. After this, many people, such as Karl Weierstrass, Hermann Schwarz, Richard Dedekind, Otto Holder, Henri Poincare, Eduard Study, and Sir Frank Macfarlane Burnet all studied the general theory of complex numbers. Augustin Louis Cauchy and Niels Henrik Able made a general theory about complex numbers accepted. August Mobius made many notes about how to apply complex numbers in geometry. All of these mathematicians helped the world better understand complex numbers, and how they are useful.

43.Kepler Conjecture(1611 AD)

In the late 1590, English nobleman and seafarer Sir Walter Raleigh set this great mathematical investigation in motion. While stocking his ship for yet another expedition, Raleigh asked his assistant, Thomas Harriot, to develop a formula that would allow him to know how many cannonballs were in a given stack simply by looking at the shape of the pile. After contemplating the question for a while, Harriot turned to one of the foremost mathematicians, physicists, and astronomers of the time, Johannes Kepler. Kepler did not reflect long, and came to the conclusion that the densest way to pack three-dimensional spheres was to stack them in the same manner that market vendors stack their apples, oranges, and melons.thus was born one of mathematics most famous problem. What is the most efficient way to pack spheres in 3D space?

The next step toward a solution was taken by Hungarian mathematician László Fejes Tóth .

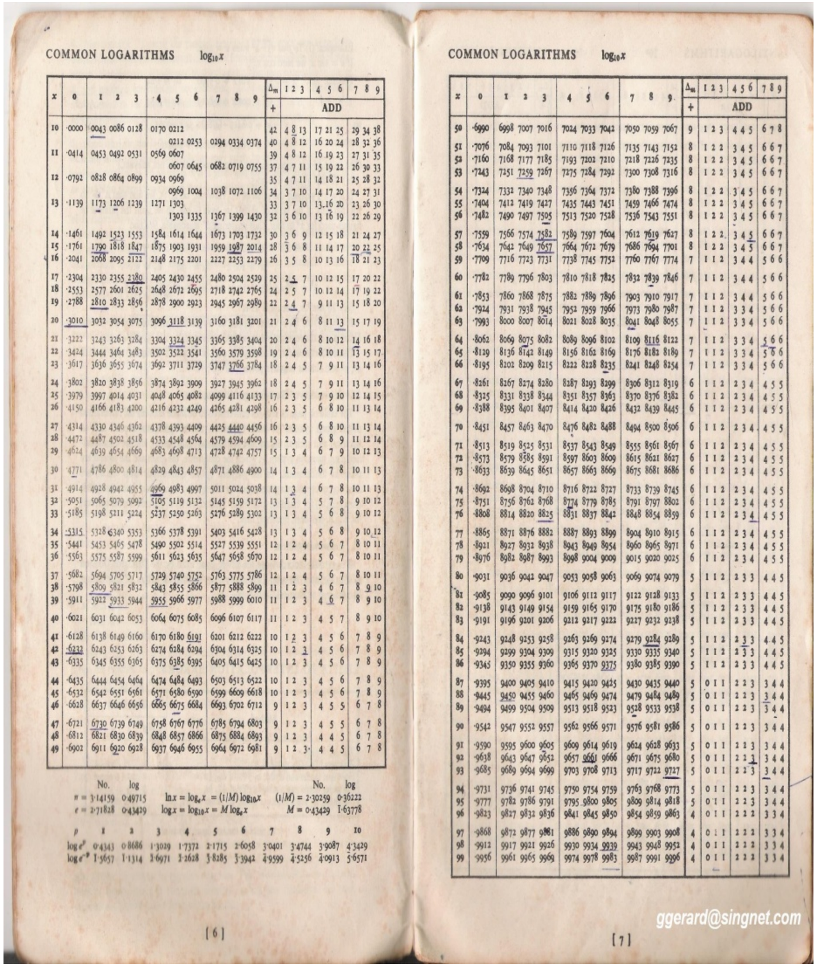
Fejes Tóth (1953) showed that the problem of determining the maximum density of all arrangements (regular and irregular) could be reduced to a finite (but very large) number of calculations. This meant that a proof by exhaustion was, in principle, possible. As Fejes Tóth realised, a fast enough computer could turn this theoretical result into a practical approach to the problem.



English mathematician Claude Ambrose Rogers (1958) established an upper bound value of about 78%, and subsequent efforts by other mathematicians reduced this value slightly, but this was still much larger than the cubic close packing density of about 74%.

Hsiang (1993, 2001) claimed to prove the Kepler conjecture using geometric methods. However Gábor Fejes Tóth (the son of László Fejes Tóth) stated in his review of the paper "As far as details are concerned, my opinion is that many of the key statements have no acceptable proofs." Hales (1994) gave a detailed criticism of Hsiang's work, to which Hsiang (1995) responded. The current consensus is that Hsiang's proof is incomplete.

44. Logarithm(1614 AD)



Logarithms were invented independently by John Napier, a Scotsman, and by Joost Burgi, a Swiss. Napier's logarithms were published in 1614; Burgi's logarithms were published in 1620. The objective of both men was to simplify mathematical calculations. This approach originally arose out of a desire to simplify multiplication and division to the level of addition and subtraction. Of course, in this era of the cheap hand calculator, this is not necessary anymore but it still serves as a useful way to introduce logarithms.

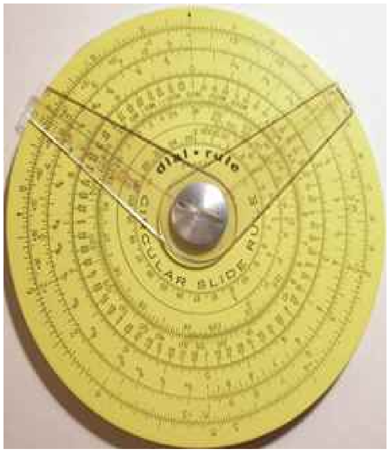
Napier's approach was algebraic and Burgi's approach was geometric. The invention of the common system of logarithms is due to the combined effort of Napier and Henry Biggs in 1624. Natural logarithms first arose as more or less accidental variations of Napier's original logarithms. The earliest natural logarithms occur in 1618. It can't be said too often: a logarithm is nothing more than an exponent. The basic concept of logarithms can be expressed as a shortcut..

Multiplication is a shortcut for Addition: 3 x 5 means 5 + 5 + 5

Exponents are a shortcut for Multiplication: 4^3 means 4 x 4 x 4

Logarithms are a shortcut for Exponents: 10^2 = 100.

45. Slide rule(1621 AD)

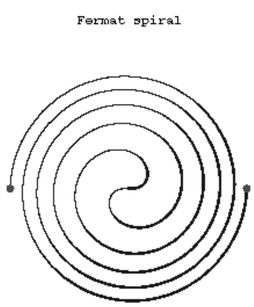


William Oughtred and others developed the slide rule in the 1600s. The slide rule is based on the work on logarithms by John Napier. Before electronic calculators were developed, slide rules were the tool used most often in science and engineering. The use of slide rules continued to grow through the 1950s and 1960s even as digital computing devices were gradually introduced; but around 1974 the electronic scientific calculator made the slide rule largely obsolete and most suppliers exited the business.

The **slide rule**, also known as a *slipstick*, is a mechanical analog computer. The slide rule is used primarily for multiplication and division, and also for "scientific" functions such as roots, logarithms and trigonometry, but does not generally perform addition or subtraction.

There are many different styles of slide rules. Generally, they are either have a linear form or that of a circle. They have a standardised set of markings (called scales). These scales are used for mathematical computations. Some slide rules have been made for specialised fields of application, for example aviationor finance. Such slide rules have special scales which are useful in the particular field of application, in addition to the common ones.

46. Fermat spiral(1636 AD)



Pierre de Fermat was one of the most brilliant and productive mathematicians of his time, making many contributions to the differential and integral calculus, number theory, optics, and analytic geometry, as well as initiating the development of probability theory in correspondence with Pascal. Pierre de Fermat was born on August 17, 1601 in Beaumont-de-Lomagne, France, and died on January 12, 1665 in Castres.

Planar transcendental curve the equation of which in polar coordinates has the form

$$\rho = a\sqrt{\phi}\ .$$

To each value of ϕ correspond two values of ρ — a positive and a negative one. The Fermat spiral is centrally symmetric relative to the pole, which is a point of inflection. It belongs to the class of so-called algebraic spirals.

They were first studied by P. Fermat (1636).

47. Fermat’s last theorem (1637 AD)

Fermat’s last theorem was actually a conjecture and remained unproved for over 300 years. It was finally proven in 1994 by Andrew Wiles, an English mathematician working at Princeton. It was always called a “theorem”, due to Fermat’s uncanny ability to propose true conjectures. Originally the statement was discovered by Fermat’s son Clement-Samuel among margin notes that Fermat had made in his copy of Diophantus’ *Arithmetica*.

Fermat followed the statement of the conjecture with the infamous teaser:

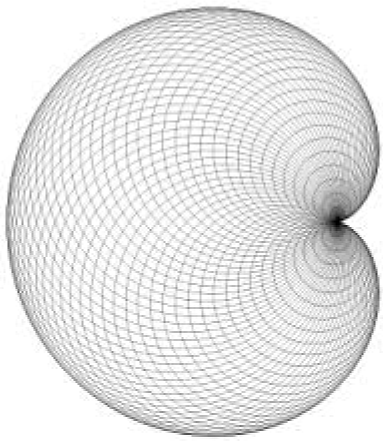
“I have discovered a truly remarkable proof which this margin is too small to contain”

Over the years, Fermat’s last theorem was proven for various sub-cases which required specific values of *n*, but no direct progress was made along these linestowards a general proof. These proofs were bittersweet victories, as each one still left an infinite number of cases unproved. Among the big names who took a crack at the theorem are Euler, Gauss, Germaine, Cauchy, Dirichlet, and Legendre.

Fermat’s last theorem was put forth by Pierre de Fermat around 1630. It statesthat the Diophantine equation

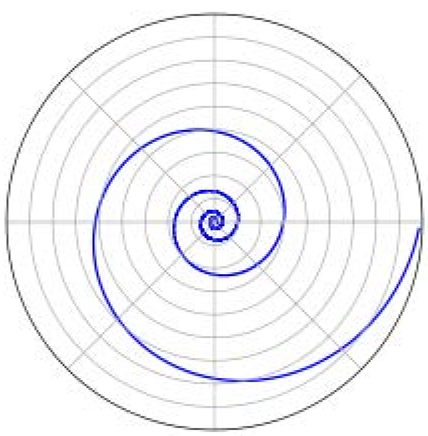
$a^n + b^n = c^n$ has no non-zero solutions for $n>2$, $(a,b,c,n \in \mathbb{N})$

48. Cardioid (1637 AD)



A cardioid is defined by the path of a point on the circumference of a circle of radius R that is rolling without slipping on another circle of radius R. Its name is derived from Greek work *kardioedides* for heart-shaped, where *kardia* means heart and *eidos*means shape, though it is actually shaped more like the outline of the cross section of an apple. The cardioid was first studied by Ole Christensen Roemer in 1674 in an effort to try to find the best design for gear teeth. However, the curve was not given its name until an Italian mathematician, Johann Castillon, used it in a paper in 1741. Since the cardioid is also a roulette, more specifically an epicycloid, and a special case of a Limacon of Pascal, it is believed that it could have originated from Etiene Pascal's studies.

49. Logarithmic Spiral (1638)



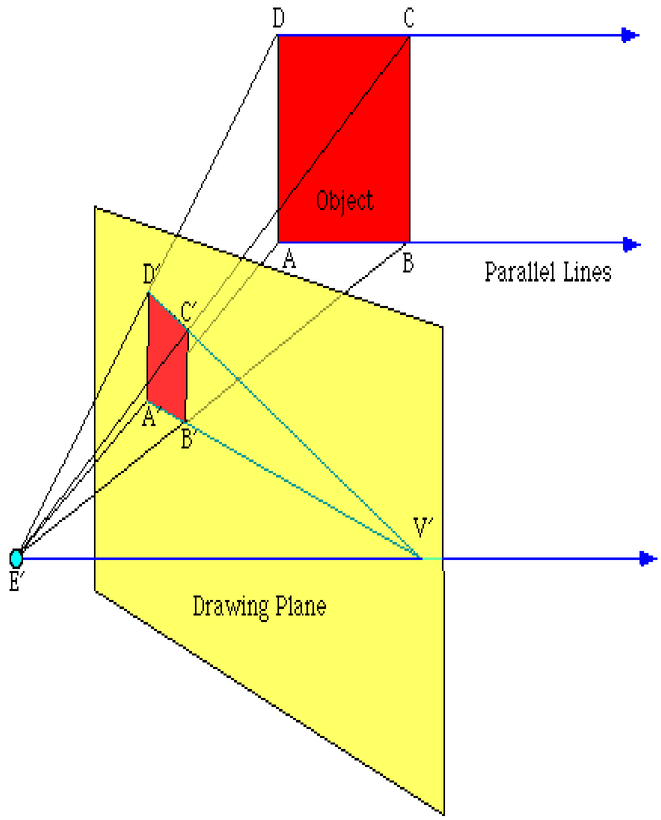
Spira mirabilis, Latin for "miraculous spiral", is another name for the logarithmic spiral. Although this curve had already been named by other mathematicians, the specific name ("miraculous" or "marvelous" spiral) was given to this curve by Jacob Bernoulli, because he was fascinated by one of its unique mathematical properties: the size of the spiral increases but its shape is unaltered with each successive curve, a property known as self-similarity. This is the spiral for which the radius grows exponentially with the angle. The logarithmic relation between radius and angle leads to the name of **logarithmic spiral** or **logistique** (in French).
The distances where a radius from the origin meets the curve are in geometric progression. The logarithmic spiral is the curve for which the angle between the tangent and the radius (the polar tangent) is a constant.

$$r = e^{a\phi}$$

Possibly as a result of this unique property, the spira mirabilis has evolved in nature, appearing in certain growing forms such as nautilus shells and sunflower heads. Jacob Bernoulli wanted such a spiral engraved on his headstone along with the phrase "Eadem mutata resurgo" ("Although changed, I shall arise the same."), but, by error, an Archimedean spiralwas placed there instead.

50. Projective Geometry (1639 AD)

The first geometrical properties of a projective nature were discovered in the third century A.D. by Pappus of Alexandria. Filippo Brunelleschi (1404–1472) started investigating the geometry of perspective in 1425.



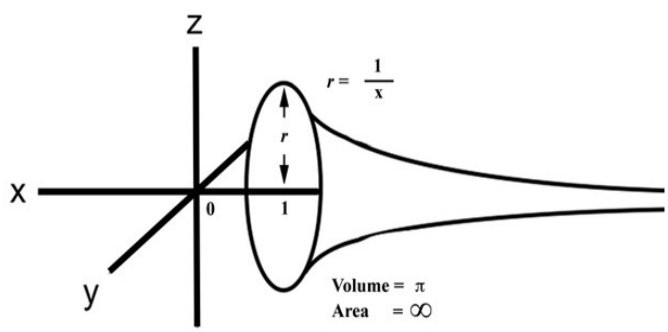
Johannes Kepler (1571–1630) and Gérard Desargues (1591–1661) independently developed the pivotal concept of the "point at infinity". Desargues developed an alternative way of constructing perspective drawings by generalizing the use of vanishing points to include the case when these are infinitely far away. He made Euclidean geometry, where parallel lines are truly parallel, into a special case of an all-encompassing geometric system. Desargues's study on conic sections drew the attention of 16-year old Blaise Pascal and helped him formulate Pascal's theorem. The works of Gaspard Monge at the end of 18th and beginning of 19th century were important for the subsequent development of projective geometry. The work of Desargues was ignored until Michel Chasles chanced upon a handwritten copy in 1845.

Meanwhile, Jean-Victor Poncelet had published the foundational treatise on projective geometry in 1822. Poncelet separated the projective properties of objects in individual class and establishing a relationship between metric and projective properties. The non-Euclidean geometries discovered shortly thereafter were eventually demonstrated to have models, such as the Klein model of hyperbolic space, relating to projective geometry. projective geometry, branch of geometry concerned with those properties of geometric figures that remain invariant under projection. The basic elements are points, lines, and planes, and the following statements are usually taken as assumptions: (1) two points lie in a unique line; (2) three points not on the same line determine a plane; (3) two lines in a plane intersect in a point; (4) two planes intersect in a line; (5) three planes not containing the same line intersect in a point. The basic elements retain their character under projection.

51. Torricelli’s trumpet(1641 AD)

Torricelli's trumpet is sometimes called Gabriel's horn and is named after Italian physicist and mathematician Evangelista Torricelli, who discovered it in 1641. He was astounded by this trumpet that seemed to be an infinitely long solid with an infinite area surface and a finite volume. Torricelli and his colleagues thought that it was a deep paradox and unfortunately did not have the tools of calculus to fully appreciate and understand the object. Today, Torricelli is remembered for the telescopic astronomy he did with Galileo and for his invention of the barometer. The name "Gabriel's horn" conjures visions of Archangel Gabriel blowing his hom to announce Judgment Day, thereby associating the infinite with the powers of God.

Torricelli’s trumpet is one famous shape to consider—a hornlike object created by revolving $f(x) = 1/x$ for $x \in$ of $[1, \infty)$ about the x-axis. Standard calculus methods can be used to demonstrate that the Torricelli’s trumpet has *finite* volume but *infinite* surface area!



Torricelli’s Trumpet

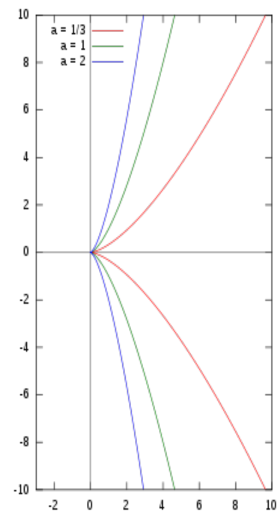
52. Pascal’s triangle (1654)

One of the most famous integer patterns in the history of mathematics is Pascal’s triangle. Blaise Pascal was the first to write a treatise about this progression in 1654, although the pattern had been known by Persian poet & mathematician Omar Khayyam as far as back as A.D. 1100 & even earlier to the mathematician of India & ancient China.

The first seven rows of Pascal’s triangle are depicted at upper right. Each number in the triangle is the sum of above two of it. Fractal figure of it is shown in figure.

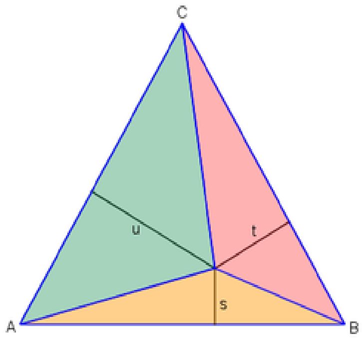
When even numbers in the triangle are replaced by dots & odd number by gaps, the resulting pattern is a fractal, with intricate repeating patterns on different size scales. These fractal figures have practical importance in that they can provide models for materials scientist to help produce new structure with novel properties. Mathematician have discussed the role of Pascal's triangle in probability theory, in the expansion of binomial theorem in the form $(x + y)^n$ and in various number theory applications for years. Mathematician Donald Knuth once indicated that there are so many relation and pattern in the Pascal's triangle. The horizontal sums of the Pascal's triangle are twice from the first one and each line is also the power exponent of 11.

53. The Length of Neile's Semi cubical Parabola (1657)



In 1657, British mathematician William Neile became the first person to "rectify," or find the arc length of, a nontrivial algebraic curve. This special curve is called a *semicubical parabola,* defined by $x^3 = ay^2$*.* When written as y = :t: $ax^{3/2}$, it is easier to see how It might have been considered "half a cubic" and hence the genesis of the term *semicubic.* A report of Neile's work appeared in British mathematician John Wallis's De *Cycloide* in 1659. Interestingly, only the arc lengths of transcendental curves, such as the logarithmic spiral and cycloid, had been calculated before 1659. Because attempts to rectify the ellipse and hyperbola were unsuccessful, some mathematicians, such as French philosopher and mathematician Rene Descartes (1596-1650), had conjectured that few curves could be rectified. Around 1687, Dutch mathematician and physicist Christian Huygens (1629-1695) showed that the semicubical parabola is a curve along which a particle may descend under the force of gravity so that it moves equal vertical distances in equal times. The semicubical parabola can also be expressed as a pair of equations: $x = t^2$ and $y = at^3$. Given this form, the length of the curve as a function of *t is (1/27)* x $(4 + 9t^2)^{3/2}$ - 8/27. In other words, the curve has this length on the interval from 0 to t. In the literature, we sometimes see Neile's parabola referred to as the curve for $y^3 = ax^2$*,* which places the cusp of the curve pointing downward along the *y-axis* instead of to the left on the x-axis.

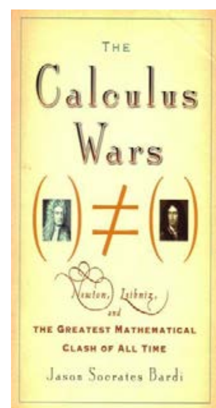
54. Viviani's theorem (1659)



The theorem is named after the Italian mathematician and scientist Vincenzo Viviani. Galileo was so impressed with the Viviani's talent that he took him into his house in Arcetri, Italy as a collaborator. According to this theorem, "If we place a point inside a equilateral triangle. From this point draw a line to each of the sides of the triangle so that these three lines are perpendicular to each side. No matter where you place the point, the sum of the perpendicular distances from the point to the sides is equal to the height of the triangle.
Researchers have found ways to extend Viviani's theorem to problems in which the point is placed outside the triangle and have also explored the application of the theorem to any regular n-sided polygon. In this case, the sum of the perpendicular distances from an interior point to the *n* sides is *n* times the apothem of the polygon. (An *apothem* is the distance from the center to a side.) The theorem can also be studied in higher dimensions.

55. Discoveries of Calculus (1665)

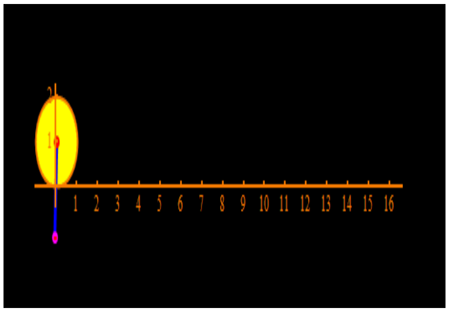
- Calculus was created by Isaac Newton, a British scientist, as well as Gottfried Leibniz, a self-taught German mathematician, in the 17th century. It has been long disputed who should take credit for inventing calculus first, but both independently made discoveries that led to what we know now as calculus. Newton discovered the inverse relationship between the derivative (slope of a curve) and the integral (the area beneath it), which deemed him as the creator of calculus. Thereafter, calculus was actively used to solve the major scientific dilemmas of the time, such as:



- calculating the slope of the tangent line to a curve at any point along its length
- determining the velocity and acceleration of an object given a function describing its position, and designing such a position function given the object's velocity or acceleration
- calculating arc lengths and the volume and surface area of solids
- calculating the relative and absolute extreme of objects, especially projectiles

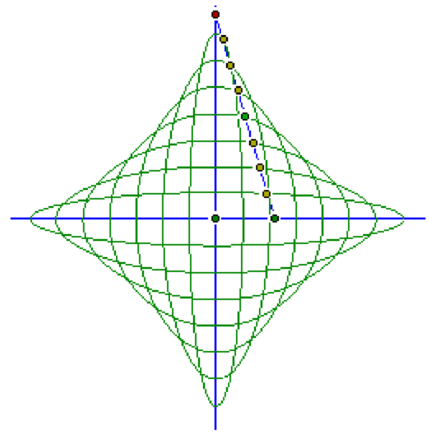
For Newton, the applications for calculus were geometrical and related to the physical world - such as describing the orbit of the planets around the sun. For Leibniz, calculus was more about analysis of change in graphs. Leibniz's work was just as important as Newton's, and many of his notations are used today, such as the notations for taking the derivative and the integral.

56. Tautochrone Problem (1673)



In the 1600s, mathematicians and physicists sought a curve that specified the shape of a special kind of ramp. In particular, objects are placed on the ramp, one at a time, and they must slide down to the very bottom, always in the same amount of time and no matter where they start on the ramp. The objects are accelerated by gravity, and the ramp is considered to have no friction.
Dutch mathematician, astronomer, and physicist Christian Huygens discovered a solution in 1673 and published it in his *Horologium oscillatoriumi (The Pendulum Clock).*
Technically speaking, the tautochrone is a cycloid- that is, a curve defined by the path of a point on the edge of a circle as the circle rolls along a straight line. The tautochrone is also called the brachistochrone when referring to the curve that gives a frictionless object the fastest rate of descent when the object slides down from one point to another. Huygens attempted to use his discovery to design a more accurate pendulum clock. The clock made use of portions of tautochrone surfaces near where the string pivoted to ensure that the string followed the optimum curve no matter where the pendulum started swinging. (Alas, the friction due to the surfaces introduced significant errors.)
The special property of the tautochrone is mentioned in *Moby Dick in* a discussion on a try-pot, a bowl used for rendering blubber to produce *oil:* "[The try-pot] is also a place for profound mathematical meditation. It was in the left-hand try-pot of the p*equod, with* the soapstone diligently circling round me, that I was first indirectly struck by the remarkable fact, that in geometry all bodies gliding along a cycloid, my soapstone, for example, will descend from any point in precisely the same time."

57. Astroid (1674)



The astroid is a curve with four cusps that is traced by a point on a circle that rolls like a gear along the inside of a larger circle. This larger circle is four times the diameter of the small circle. The astroid is notable for the diversity of famous mathematicians who researched its intriguing properties. The curve was first studied by the Danish astronomer Ole Rl'lmer in 1674, as a result of his quest for gear teeth with more useful shapes. Swiss mathematician Johann Bernoulli (1691), German mathematician Gottfried Leibniz (1715), and French mathe matician Jean d'Alembert (1748) all became fascinated by the curve.
The astroid has the equation $x^{2/3} + y^{2/3} = R^{2/3}$, where R is the radius of the stationary outer circle, and *R/4* is the radius of the inner rolling circle. The length of the astroid *is* 6R, and the area is $3\pi R^2/8$. Interestingly, its 6R circumference has no dependence on π, despite the involvement of circles that are used for generating the astroid.

In 1725, mathematician Daniel Bernoulli discovered that an astroid is also traced by an inner circle that has ¾ the diameter of the fixed circle. In other words, this traces out the same curve as the inner circle with only ¼ the diameter of the larger one.
In physics, the Stoner-Wolfforth astroid curve is used to characterize various
Properties of energy and magnetism. U.S. Patent 4,987,984 describes the use of an asteroid for mechanical roller clutches: "The astroid curve provides the same good dispersal of stresses that the equivalent circular arc would, hut removes less cam race material, giving a stronger structure."
Interestingly, tangent lines along the astroid curve, when extended until they touch the x- and y-axis, all have the same length. You can visualize this by imagining a ladder leaning at all possible angles against a wan, which traces out a portion of the astroid curve.

58. L'Hopital's Analysis of the Infinitely Small (1696)

Suppose that we have one of the following cases,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0} \qquad \text{OR} \qquad \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0} \qquad \text{OR} \qquad \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty}$$

where *a* can be any real number, infinity or negative infinity. In these cases we have,

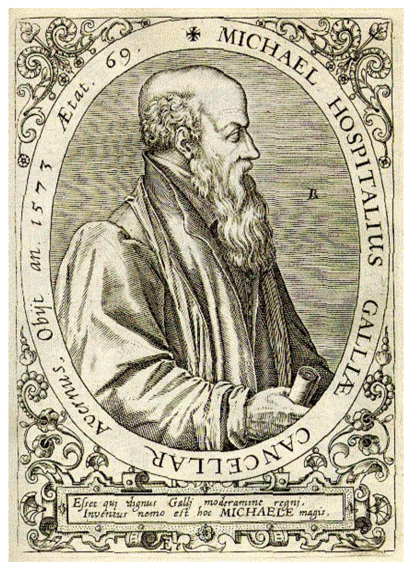
$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

So, L'Hospital's Rule tells us that if we have an indeterminate form 0/0 or ∞/∞ all we need to do is differentiate the numerator and differentiate the denominator and then take the limit.

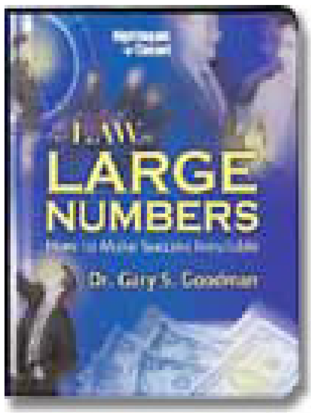
In 1696, French mathematician Marquis de I'Hapital published Europe's first calculus textbook, *Analyse des infiniment petit., POUT I'intelligence* des *lignes courbes (Analysis* of *the Infinitely Small, for the Understanding of Calculus)*.

In the early 1690s, I'Hopital hired Johann Bernoulli to teach him calculus.

Aside from his textbook, I'Hopital is known for the rule of calculus, included in his book, for calculating the limiting value of a fraction whose numerator and denominator either both approach zero or both approach infinity. He initially had planned a military career, but poor eyesight caused him to switch to mathematics. Today, we know that I'Hapital, in 1694, paid Bernoulli 300 francs a year to tell him of his discoveries, which I'Hapital described in his book. In 1704, after I'Hopital's death, Bernoulli began to speak of the deal and claimed that many of the results in *Analysis* of *the Infinitely Small* were due to him.



59. Law of Large Numbers (1713)

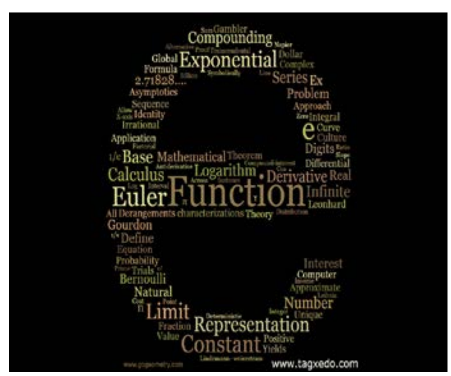


In 1713, Swiss mathematician Jacob Bernoulli's proof of his Law of Large Numbers (LLN) was presented in a posthumous publication, Ars *Conjectandi (The Art* of *Conjecturing).* The LLN is a theorem in probability that describes the long-term stability of a random variable. For example, when the number of observations of an experiment (such as the tossing of a coin) is sufficiently large, then the proportion of an outcome (such as the occurrence of heads) will be close to the probability of the outcome, for example 0.5. Stated more formally, given a sequence of independent and identically distributed random variables with a finite population mean and variance, the average of these observations will approach the theoretical population mean

In Ars *Conjectandi,* Bernoulli estimates the proportion of white balls in an urn filled with an unknown number of black and white balls. By drawing balls from the urn and "randomly" replacing a ball after each draw, he estimates the proportion of white balls by the proportion of balls drawn that are white. By doing this enough times, he obtains any desired accuracy for the estimate. Bernoulli writes, "If observations of all events were to be continued throughout all eternity (and, hence, the ultimate probability would tend toward perfect certainty), everything in the world would be perceived to happen in fixed ratios .... Even in the most accidental... occurrences, we would be bound to **recognize . .. a certain fate."**

60. Euler's Number, e (1727)

The number e, which is approximately equal to 2.71828, can be calculated in many ways. For example, it is the limit value of the expression (1+1/n*)* raised to the nth power, when n increases indefinitely. Although mathematicians like Jacob Bernoulli and Gottfried Leibniz were aware of the constant, Swiss mathematician Leonhard Euler was among the first to extensively study the number, and he was the first to use the symbol e in letters written in 1727. In 1737, he showed that e is irrational-that is, it cannot be expressed as a ratio of two integers. In 1748, he calculated 18 of its digits, and today more than 100,000,000,000 digits of e are known.



61. Stirling's Formula (1730)

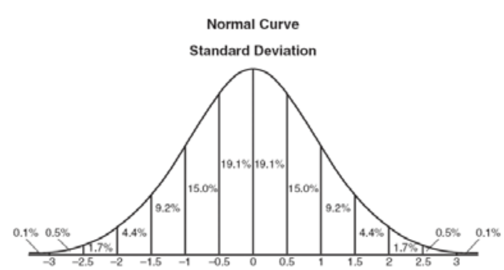
Stirling's Inequality for n!

$$\sqrt{2\pi}\, n^{n+\frac{1}{2}} e^{-n+\frac{1}{12n+1}} < n! < \sqrt{2\pi}\, n^{n+\frac{1}{2}} e^{-n+\frac{1}{12n}}$$

These days, factorials are everywhere in mathematics. For non-negative integers *n, "n* factorial" (written as *n!),* is the product of all positive integers less than or equal to *n.*
For example, 4! = I x 2 x 3 x 4 = 24. The notation n! was introduced by French mathematician Christian Kramp in 1808. Factorials are important in combin -atorics, for example, when determining the number of different ways of arranging objects in a sequence. They also occur in number theory, probability, and calculus. Because factorial values grow so large (for example, 70! is greater than 10^{100}, and 25,206! is greater than $10^{100,000}$), convenient methods for approximating large factorials are extremely useful. Stirling's formula, n! = $\sqrt{2\pi}\ e^{-n} n^{n+1/2}$, *provides* an accurate estimate for *n* factorial. Here, the ≈ symbol means "approximately equal to," and e and π are the mathematical constants e ≈ 2.71828 and π ≈ 3.14159. For large values of *n,* this expression results in an even simpler-looking approximation,ln(n!) ≈ nln(n) - n, which can also be written as n! ≈ $n^n e^{-n}$. In 1730, Scottish mathematician James Stirling presented his approximation for the value of nl in his most important work, *Methodus Differentialis.* Stirling began his career in mathematics amidst political and religious conflict.

62. Normal Distribution Curve (1733)

In 1733, French mathematician Abraham de Moivre was the first to describe the normal distribution curve, or law of errors, in *Approximatio ad summam tenninorum binomii* $(a+b)^n$ *in seriem expansi* ("Approximation to the Sum of the Terms of a Binomial $(a+b)^n$ Expanded as a Series").



Throughout his life, de Moivre remained poor and earned money on the side by playing chess in coffeehouses.
The normal distribution-also called the Gaussian distribution, in honor of Carl Friedrich Gauss, who studied the curve years later- represents an important family of continuous probability distributions that are applied in studies of population demographics, health statistics, astronomical measurements, heredity, intelligence, insurance statistics etc.

The normal distribution is defined by two key parameters, the mean (or average) and the standard deviation, which quantifies the spread or variability of the data. The normal distribution, when graphed, is often called the *bell curve* because of its symmetric be like shape with values more concentrated in the middle than in the tails at the sides of the curve.

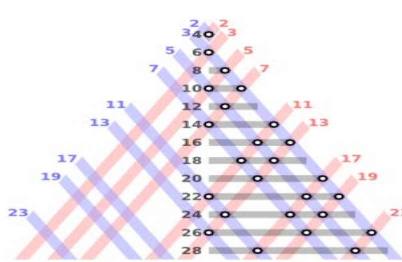
63. Euler-Mascheroni Constant (1735)

The Euler-Mascheroni constant, denoted by the Greek letter γ, has a numerical value of 0.5772157 . . .. This number links the exponentials and logarithms to number theory, and it is defined by the limit of (I +*1/2* +1/3+ ... + *1/n* - log *n)* as *n* approaches infinity. The reach of γ is far and wide, as it plays roles in such diverse areas as infinite series, products, probability, and definite integral representations. For example, the average number of divisors of all numbers from I to *n* is very close to In *n* + 2γ – 1. While we presently know π to 1,241, I 00,000 ,000 decimal places, in 2008, only about 10,000,000,000 places of γ were known

The evaluation of γ is considerably more difficult than π. Here are the first few digits: 0.5772 15664901532860606512090082402431042159335939 92 . ... Swiss mathematician Leonhard Euler discussed γ in a paper. *"De Progressionibus harmonicis observationes"* ("Observations about Harmonic Progressions"), published in 1735, but he was only able to calculate it to six decimal places at the time. In 1790, Italian mathematician and priest Lorenzo Mascheroni computed additional digits. Today, we don't know if the number can be expressed as a fraction (in the way that a number like O.l428571428571...can be expressed as In). Julian Havil, who devoted an entire book to γ, tells of stories in which the English mathematician C. H. Hardy offered to give up his Savilian Chair at Oxford to anyone who proved γ could not be expressed as a fraction.

64. Goldbach Conjecture (1742)

The most challenging problems in mathematics are among the easiest and simplest to state. In 1742, Prussian historian and mathematician Christian Goldbach conjectured that every integer greater than 5 can be written as the sum of three prime numbers, such as 21 = 11 + 7 + 3. (A prime number is a number larger than 1, such as 5 or 13, that is divisible only by itself or 1.)



As re-expressed by Swiss mathematician Leonhard Euler, an equivalent conjecture (called the "strong" Goldbach conjecture) asserts that all positive even integers greater than 2 can be expressed as the sum of two primes. In order to promote the novel *Uncle Petros and Goldbach's Conjecture,* publishing giant Faber and Faber offered a $1,000,000 prize to anyone who proved Goldbach's conjecture between March 20, 2000, and March 20, 2002, but the prize went unclaimed, and the conjecture remains open. In 2008, Tomas Oliveira e Silva, a researcher at the University of Aveiro, Portugal, ran a distributed computer search that has verified the conjecture up to 12. 10^{17} Of course, no amount of computing power can confirm the conjecture for every number; thus, mathematicians hope for an actual proof that Goldbach's intuibon was right. In 1966, Chen Jing-Run, a Chinese mathematician, made some progress when he proved that every sufficiently large even number is the sum of one prime, plus a number that is the product of at most two primes. So, for example, 18 is equal to 3 + (3 X 5). In 1995, french mathematician Olivier Raman! showed that every even number greater than or equal to 4 is the sum of at most six primes.

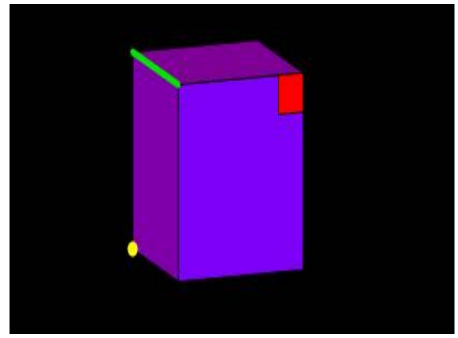
65. Agnesi's *Instituzioni Analitiche (1748)*

Italian mathematician Maria Agnesi is the author of *Instituzioni analitiche (Analytical Institutions),* the first comprehensive textbook that covered both differential and integral calculus, and the first surviving mathematical work written by a woman. Dutch mathematician Dirk Ian Struik referred to Agnesi as "the **first important woman mathematician** since Hypatia (A.D. fifth century)."

Agnesi was a child prodigy, speaking at least seven languages by age 13. For much of her life, she avoided social interactions and devoted herself entirely to the study of mathematics and religion. Clifford Truesdell writes, "She did ask her father's permission to become a nun. Horrified that his dearest child should desire to leave him, he begged her to change her mind." She agreed to continue living with her father so long as she could live in relative seclusion. The book also includes a discussion of the cubic curve now known as the Witch of Agnesi and expressed as $y = 8a^3/(x^2+4a^2)$. Agnesi spent all her money on helping the poor and she died in total poverty in a poorhouse.

66. Euler's Formula for Polyhedra (1751)

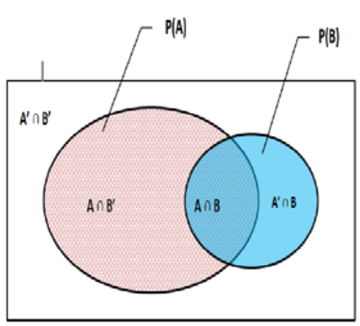
Euler's formula for polyhedra is considered to be one of the most beautiful formulas in all of mathematics and one of the first great formulas of topology-the study of shapes and their interrelationships. A survey conducted of *Mathematical Intelligencer* readers ranked the formula as the second most beautiful formula in history, second to Euler's $e^{i\pi}+1 = 0$ discussed in the entry Euler's Number, e (1727).



In 1751, Swiss mathematician and physicist Leonhard Euler discovered that any convex polyhedron (an object with Hat faces and straight edges), with V vertices, E edges, and F faces, satisfies the equation $V - E + F = 2$. A polyhedron is *convex* if it has no indentations or holes, or more formally, if every line segment connecting interior points is entirely contained within the interior of the figure.

Interestingly, around 1639, Rene Descartes discovered a related polyhedral formula that may be converted to Euler's formula through several mathematical steps. The polyhedron formula was later generalized to the study of networks and graphs, and to help mathematicians understand a wide range of shapes with holes and in higher dimensions. Sadly, Euler went blind toward the end of his life. However, British science writer David Darling notes, "the quantity of his output seemed to be inversely proportional to the quality of his sight, because his rate of publication increased after he became almost totally blind in 1766."

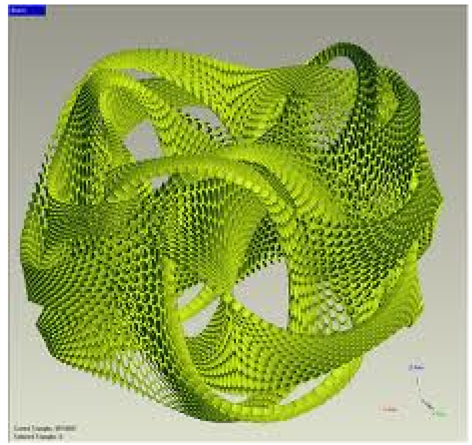
67. Bayes' Theorem (1761)



Bayes' theorem, formulated by British mathematician and Presbyterian minister Thomas Bayes in 1761. *Conditional probability* refers to the probability of some event A, given the occurrence of some other event B, written as P(A/B). Bayes' theorem states: $P(A/B) = [P(B/A) \times P(A)]/P(B)$. Here, P(A) is called the prior probability of A because it is the probability of event A without taking into account anything we know about B. P(B/A) is the conditional probability of B given A. P(B) is the prior probability of B.

68. Minimal Surface (1774)

Imagine withdrawing a flat wire ring from soapy water. Because the ring contains a disk-shaped soap film that has less area than other shapes that hypothetically may have formed, mathematicians call the surface a minimal surface. More formally, a finite minimal surface is often characterized as having the smallest possible area bounded by a given closed curve or curves.



The mean curvature of the surface is zero. The mathematician's quest for minimal surfaces and proofs of their minimality has lasted for more than two centuries. Minimal surfaces with bounding curves that twist into the third dimensions can be both beautiful and complicated. In 1744, Swiss mathematician Leonhard Euler discovered the catenoid, the first example of a minimal surface beyond mere trivial examples like circular areas. In 1776, French geometer Jean Meusnier discovered the helicoid minimal surface. Another minimal surface wasn't found until 1873 by German mathematician Heinrich Scherk. The same year, the Belgian physicist Joseph Plateau performed experiments that led him to conjecture that soap films always form minimal surfaces. "Plateau's problem" deals with the mathematics required to prove this to be true. (Plateau went blind as a result of staring into the sun for 25 seconds in an experiment dealing with vision physiology.) More recent examples include Costa's minimal surface, which was first described mathematically in 1982 by Brazilian mathematician Celso Costa. Computers and computer graphics now play a significant role in helping mathematicians construct and visualize minimal surfaces.

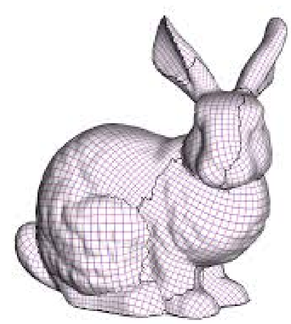
69. Sangaku Geometry (1789)

A tradition known as Sangaku, or "Japanese temple geometry," arose during Japan's period of isolation from the West, roughly between 1639 and 1854. Mathematicians, farmers, samurai, women, and children solved difficult geometry problems and inscribed the solutions on tablets. These colorful tablets were then hung under the roofs of the temples. More than 800 tablets have survived, and many of them feature problems concerning tangent circles

As one example, consider the figure on the opposite page, a late Sangaku tablet from 1873 created by an 13-year-old boy named Kinjiro Takasaka. The illustration shows a fan, which is one-third of a complete circle. Given the diameter d1 of the yellow-shaded circle, what is the diameter d2 of the green shaded circle? The answer is $d_2 \approx d_1(\sqrt{3072} + 62)/193$.
In 1789, Japanese mathematician Fujita Kagen published *Shimpeki Sampo (Mathematical Problems Suspended before the Temple),* the first collection of Sangaku problems. The oldest surviving tablet dates from 1683, although other historical documents refer to examples from as early as 1668. Most of the Sangaku are strangely different from typical geometry problems found in textbooks because the Sangaku aficionados were usually obsessed with circles and ellipses. Some of the Sangaku problems are so difficult that physicist Tony Rothman and educator Hidetoshi Fukagawa write, "Modem geometers invariably tackle them with advanced methods, including calculus and affine transform ations." However, by avoiding calculus. Sangaku problems were, in principle, sufficiently simple that children could solve them with some effort. Chad Boutin writes, "Perhaps it's not surprising that Sudoku -the number puzzles that everyone seems to be working on these days-first became popular in Japan before spreading across the ocean.

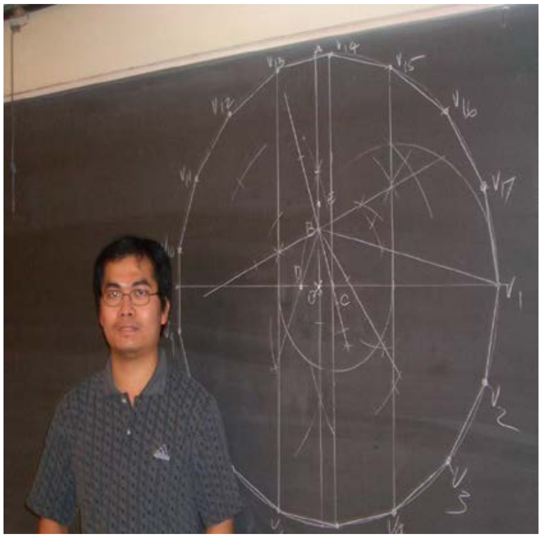
70. Least Squares (1795)

Least squares is a mathematical procedure for finding the "best-fitting" curve for a given set of data points by minimizing the sum of the squares of the offsets of the points from the curve.
In 1795, German mathematician and scientist Carl Friedrich Gauss, at the age of 18, began to develop least-squares analysis. He demonstrated the vaille of his approach in 1801, when he predicted the future location of the asteroid Ceres.



As background, the Italian astronomer Giuseppe Piazzi (1746-1826) had originally discovered Ceres in 1800, but the asteroid later disappeared behind the sun and could not be relocated. Austrian astronomer Franz Xavier von Zach (1754-1832) noted that "without the intelligent work and calculations of Doctor Gauss, we might not have found Ceres again." Interestingly, Gauss kept his methods a secret to maintain an advantage over his contemporaries and to enhance his reputation. Later in his life, he sometimes published scientific results as a cipher, so that he could always prove that he had made various discoveries before others had. Gauss finally published his secret least-squares method in 1809 in his Theory of *the Motion* of *the Heavenly Bodies.*

71. Constructing a Regular Heptadecagon (1796)

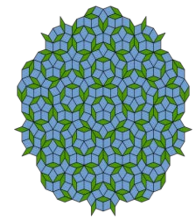


In 1796, when Gauss was still a teenager, he discovered a way to construct a regular 17-sided polygon, also known as a heptadecagon, using just a straightedge and compass. He published the result in his monumental 1801 work, *Disquisitiones Arithmeticae (Arithmetic Disquisitions).* Gauss's construction was very significant because only failed attempts had been made since the time of Euclid. For more than 1,000 years, mathematicians had known how to construct, with a compass and straightedge, regular n-gons in which n was a multiple of 3, 5, and powers of 2.

Gauss was able to add more polygons to this list, namely those with a prime number of sides of the form $2^{(2n)}+ I$, where n is an integer. We can make a list of the first few such numbers: $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$ & $F_4 = 65{,}537$. (Numbers of this form are also known as Fermat numbers, and they are not necessarily prime.) A 257-gon was constructed in 1832. When he was older, Gauss still regarded his 17-gon finding as one of his greatest achievements and he asked that a regular 17-gon be placed on his tombstone. According to legend, the stonemason declined, stating that the difficult construction would essentially make the 17-gon look like a circle. The year 1796 was an auspicious year for Gauss, when his ideas gushed like a fountain from a fire hose. Aside from solving the heptadecagon construction (March 30), Gauss invented modular arithmetic and presented his quadratic reciprocity law (April 8) and the prime number theorem (May 31). He proved that every positive integer is represented as a sum of at most three triangular numbers (July 10). He also discovered solutions of polynomials with coefficients in finite fields (October 1).

72. Fundamental Theorem of Algebra (1797)

The Fundamental Theorem of Algebra (FTA) is stated in several forms, one of which is that every polynomial of degree n≥1, with real or complex coefficients, has *n* real or complex roots. In other words, a polynomial *P(x)* of degree *n* has *n* values x_i (some of which are possibly repeated) for which *P(Xi)* = 0.



As back ground, polynomial equations of degree *n* are of the form $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0 = 0$ where $a_n \neq 0$. This theorem is notable, in part, because of the sheer number of attempts at proving it through history. German mathematician Carl Friedrich Gauss is usually credited with the first proof of the FTA, discovered in 1797. In his doctoral thesis, published in 1799, he presented his first proof, which focused on polynomials with real coefficients, and also on his objections to the other previous attempts at proofs. By today's standards, Gauss's proof was not rigorously complete, because he relied on the continuity of certain curves, but it was a significant improvement over all previous attempts at a proof. His fourth proof was in the last paper he ever wrote, which appeared in 1849, exactly 50 years after his dissertation. Jean-Robert Argand (1768-1822) also published a rigorous proof of the Fundamental Theorem of Algebra in 1806 for polynomials with complex coefficients

73. Gauss's DisquisitionesArithmeticae (1801)

In the Disquisitiones, Gauss introduced the notion of congruence and in so doing unified number theory." Gauss published this monumental work at the age of 24. The Disquisitiones involves modular arithmetic, which relies on congruency relationships. Two integers p and q are "congruent modulo the integer s" if and only if (p - q) is evenly divisible by s. Such congruence is written as $p \equiv q \pmod{s}$. Using this compact notation, Gauss restated and proved the famous quadratic reciprocity theorem, which was incompletely proven several years earlier by French mathematician Adrien- Marie Legendre (1752-1833).

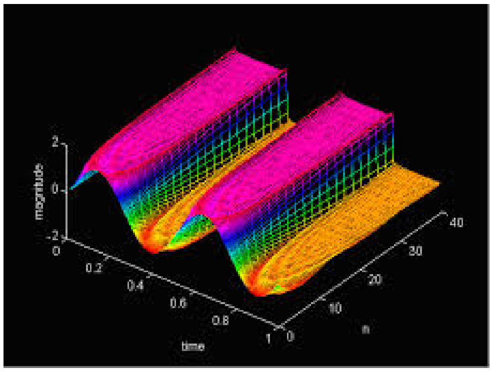
Gauss devoted an entire section of his book to his proof of this theorem. He considered this beloved theorem of quadratic reciprocity to be the "golden theorem" or the "gem of arithmetic," which so enthralled Gauss that he went on to provide eight separate proofs over his lifetime. In Disquisitiones, Gauss's approach to providing theorems, followed by proofs, corollaries, and examples was used by subsequent authors.

74.Three-Armed Protractor (1801)

In the seventeenth century, protractors began to be used as stand-alone instruments rather than as parts of other devices, when sailors used them on ocean maps. In 1801, Joseph Huddart, an English naval captain invented the three-armed protractor for plotting the position of a boat on navigation maps. This kind of protractor makes use of two outer arms that may rotate with respect to a fixed central arm. The two rotating arms may be clamped so that they can be set at fixed angles.

In 1916, the United States Hydrographic Office explained the use of his protractor: "To plot a position, the two angles observed between the three selected [known] objects are set on the instrument, which is then moved over the chart until the three beveled edges pass respectively and simultaneously through the three objects. The center of the instrument will then mark the ship's position which may be pricked on the chart or marked with a pencil point through the center hole."

75. Fourier Series (1807)



Fourier series are useful in countless applications today, ranging from vibration analysis to image processing-virtually any field in which a frequency analysis is important. Before French mathematician Joseph Fourier discovered his famous series, he accompanied Napoleon on his 1789 expedition of Egypt, where Fourier spent several years studying Egyptian artifacts. Fourier's research on the mathematical theory of heat began around 1804 when he was back in France, and in 1807 he had completed his important memoir *On the Propagation* of *Heat in Solid Bodies.* One of his interests was heat diffusion in different shapes.

For these problems, researchers are usually given the temperatures at points on the surface, as well as at its edges, at time *t* = O. Fourier introduced a series with sine and cosine terms in order to find solutions to these kinds of problems. More generally, he found that any differentiable function can be represented to arbitrary accuracy by a sum of sine and cosine functions, no matter how bizarre the function may look when graphed. British physicist Sir James Jeans (1877-1946)
remarked, "Fourier's theorem tells us that every curve, no matter what its nature may be, or in what way it was originally obtained, can be exactly reproduced by superposing a sufficient number of simple harmonic curves-in brief, every curve can be built up by piling up waves."

76. Laplace's Theorie Analytique des Probabilites (1812)

The first major treatise on probability that combines probability theory and calculus was French mathematician and astronomer Pierre-Simon Laplace's Theorie Analytique des Probabilites (Analytical Theory of Probabilities). Probability theorists focus on random phenomena. Although a single roll of the dice may be considered a random event, after numerous repetitions, certain statistical patterns become apparent, and these patterns can be studied and used to make predictions.

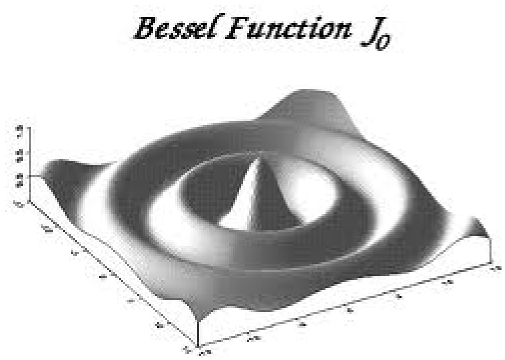
The first edition of Laplace's Theorie Analytique was dedicated to Napoleon Bonaparte and discusses methods of finding probabilities of compound events from component probabilities. The book also discusses the method ofleast squares and Buffon's Needle and considers many practical applications. To explain how probabilistic processes can yield predictable results, Laplace asks readers to imagine several urns arranged in a circle. One urn contains only black balls, while another contains only white balls. The other urns have various ball mixtures. If we withdraw a ball, place it in the adjacent urn, and continue around the circle, eventually the ratio of black to white balls will be approximately the same in all of the urns.

77. Bessel Functions (1817)

In the Sturm-Liouville Boundary Value Problem, there is an important special case called Bessel's Differential Equation which arises in numerous problems, especially in polar and cylindrical coordinates. Bessel's Differential Equation is defined as:

$$x^2y'' + xy' + \left(x^2 - n^2\right)y = 0$$

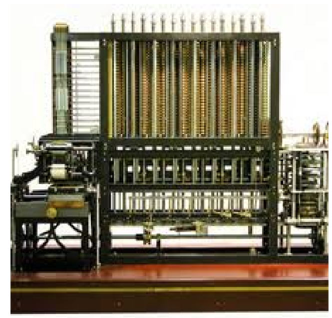
Where n is a non-negative real number. The solutions of this equation are called Bessel Functions of order



Although the order can be any real number, the scope of this section is limited to non-negative integers, i.e., unless specified otherwise.German mathematician Friedrich Bessel, who had no formal education after the age of 14, developed Bessel functions in 1817 for use in his studies of the motion of planets moving under mutual gravitation. Bessel had generalized the earlier findings of mathematician Daniel Bernoulli (1700-1782).Bessel functions are solutions to specific differential equations, and when graphed, the functions resemble rippling, decaying sinusoidal waves.In 2006, researchers at Japan's Akishima Laboratories and Osaka University relied on Bessel function theory to create a device that uses waves to draw actual text and pictures on the surface of water. The device, called AMOEBA (Advanced Multiple Organized Experimental Basin), consists of 50 water wave generators encircling a cylindrical tank 1.6 meters in diameter and 30 cm deep. AMOEBA is capable of spelling out the entire Roman alphabet. Each picture or letter remains on the water surface only for a moment, but they can be produced in succession every few seconds.

78. Babbage Mechanical Computer (1822)

Charles Babbage was an English analyst, statistician, and inventor who was also interested in the topic of religious miracles. Babbage is often considered the most important mathematician-engineer involved **in the prehistory of computers. In particular, he is famous for conceiving an enormous** hand-cranked mechanical calculator, an early progenitor of our modem computers.



Babbage thought the device would be most useful in producing mathematical tables, but he worried about mistakes that would be made by humans who transcribed the results from its 31 metal output wheels. Babbage's Difference Engine, begun in 1822 but never completed, was designed to compute values of polynomial functions, using about 25,000 mechanical parts. He also had plans to create a more general-purpose computer, the Analytical Engine, which could be programmed using punch cards and had separate areas for number storage and computation. Estimates suggest that an Analytical Engine capable of storing 1,000 50-digit numbers would be more than 100 feet (about 30 meters) in length. Ada Lovelace, the daughter of the English poet Lord Byron, gave specifications for a program for the Analytical Engine. Although Babbage provided assistance to Ada, many consider Ada to be the first computer programmer.

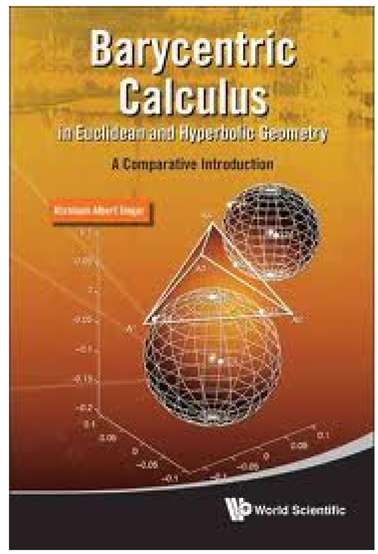
79. Cauchy's *Le Calculus Infinitesimal (1823)*

In his 1823 *Resume des lecons sur* le *calcul infinitesimal (Resume of Lessons on Infinitesimal Calculus),* the prolific French mathematician Augustin Cauchy provides a rigorous development of calculus and a modern proof of the Fundamental Theorem of Calculus, which elegantly unites the two major branches of calculus (differential and integral) into a single framework. Cauchy begins his treatise with a clear definition of the derivative. His mentor, French mathematician Joseph-Louis Lagrange (1736-1813), thought in terms of graphs of curves and considered the derivative a tangent to a curve. In order to determine a derivative, Lagrange would search for derivative formulas as necessary.

Similarly, by clarifying the notion of the integral in calculus, Cauchy demonstrated the Fundamental Theorem of Calculus, which establishes a way in which we can compute the integral of *f(x)* from x = a to x = *b* for any continuous function f. More particularly, the Fundamental Theorem of Calculus states that if *f is* an integrable function in the interval *[a,* b], and if *H(x)* is the integral of *f(x)* from a to x ≤ *b,* then the derivative of *H(x)* is identical to *f(x).* In other words, *H'(x) = f(x).*

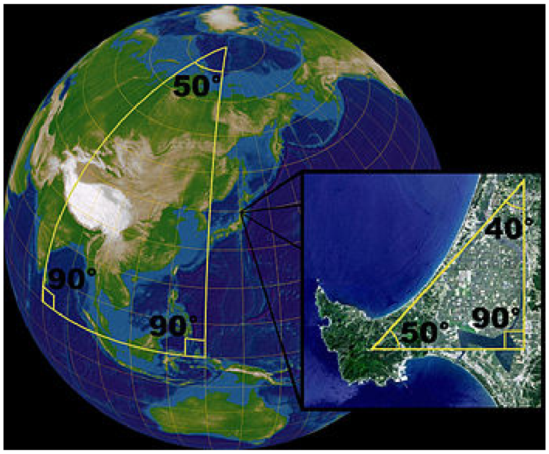
80. Barycentric Calculus (1827)

German mathematician August Ferdinand Mobius, famous for his one-sided loop called the Mobius strip, also made a major contribution to mathematics with his barycentric calculus, a geometrical method for defining a point as the center of gravity of certain other points to which coefficients or weights are ascribed. Mobius's *barycentric coordinates* (or *barycentrics) are* as coordinates with respect to a reference triangle. These coordinates are usually written as triples of numbers, which can be visualized as corresponding to masses placed at the vertices of the triangle. In this way, these masses determine a point, which is the geometric centroid of the three masses.



The new algebraic tools, developed by Mobius in his 1827 book *Der Barycentrische Calcul (The Barycentric Calculus),* have since turned out to have wide application. This classic book also *discusses* related topics in analytical geometry such as projective transformations. The word *barycentric* is derived from the Greek *barys* for "heavy" and refers to the center of mass. Mobius understood that several weights positioned along a straight stick can be replaced by a single weight at the stick's center of mass. From this simple principle, he constructed a mathematics system in which numerical coefficients are assigned to every point in space.

81. Non-Euclidean Geometry (1829)



Since the time of Euclid (c. 325-270 B.C.), the so-called parallel postulate seemed to reasonably describe how our three-dimensional world works. According to this postulate, given a straight line and a point not on that line, only one straight line through the point exists that never intersects the original line. Over time, the formulations of non-Euclidean geometry, in which this postulate does not hold, have had dramatic consequences. Einstein's General Theory of Relativity represents space-time as a non-Euclidean geometry in which space-time actually warps, or curves, near gravitating bodies such as the sun and planets.

This can be visualized by imagining a bowling ball place a marble into the depression formed by the sideways push, it would orbit the bowling ball for a while, like a planet orbiting the sun.
In 1829, Russian mathematician Nicolai Lobachevsky published *On the Principles of Geometry,* in which he imagined a perfectly consistent geometry that results from assuming that the parallel postulate is false. Several years earlier, Hungarian mathematician Janos Bolyai had worked on a similar non-Euclidean geometry, but his publication was delayed until 1932. In 1854, German mathematician Bernhard Riemann generalized the findings of Bolyai and Lobachevsky by showing that various non-Euclidean geometries are possible, given the appropriative number of dimensions.

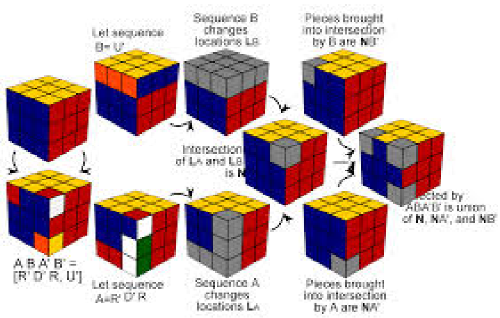
82. Mobius Function (1831)

In 1831, August Mobius introduced his exotic Mobius function, today written as $\mu(n)$. To understand the function, imagine placing all the integers into just one of three large mailboxes. The first mailbox is painted with a big 0, the second with + 1 and a third with -1. In mailbox 0, Mobius places multiples of square numbers (other than 1), including (4, 8, 9, 12, 16, 18 ....). A *square number* is a number such as 4, 9, or 16 that is the square of another integer. For example, μ(12) = 0, because 12 is a multiple of the square number 4 and is thus placed in mailbox "0." In the -1 mailbox, Mobius places any number that factors into an odd number of distinct prime numbers. For example, 5x 2 x 3 = 30, so 30 is in this list because it has these three prime factors. All prime numbers are also on this list because they only have one prime factor, themselves. Thus, μ(29) = - I and μ(30) = - 1.

The probability that a number falls in the – 1 mailbox turns out to be $3/\pi^2$ -the same probability as for falling in the +1 mailbox. Let's further consider the + 1 mailbox. in which Mobius places all the numbers, such as 6, that factor into an even number of distinct primes (2 X 3 = 6). For completeness, Mobius put 1 into this bin. Numbers in this mailbox include (I, 6, 10, 14, I5, 21, 22, .. .). The first 20 terms of the wonderful Mobius function are $\mu(n)$ = (1, - 1, - 1, 0, -1, 1, -1,0,0, 1, - 1, 0, -1, 1, 1, 0, - 1, 0, - 1, O).

Amazingly, scientists have found practical uses of the Mobius function in various physical interpretations of subatomic particle theory. The Mobius function is also fascinating because almost everything about its behavior is unsolved and because numerous elegant mathematical identities exist that involve $\mu(n)$.

83. Group Theory (1832)

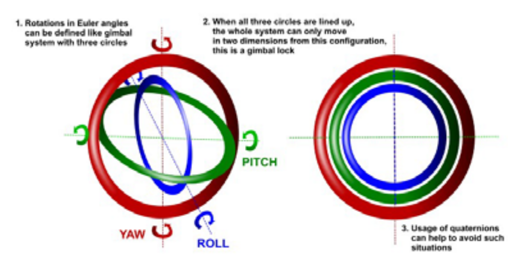


French mathematician Evariste Calois was responsible for Calois theory, an important branch of abstract algebra, and famous for his contributions to group theory, which concerns the mathematical study of symmetry in 1832. He produced a method of determining when a general equation could be solved by radicals, thereby, in essence giving a kick start to modern group theory. "In 1832, he was killed by a pistol shot. He was not yet 21.

Calois who laid the foundations of modern group theory in a sad letter that he wrote to a friend the night before his fatal duel." One key aspect of a group is that it is a set of elements with an operation that combines any two of its elements to form a third element within that set. A geometrical object can be characterized by a group called a symmetry *group* that specifies the symmetry features of the object. This group contains a set of transformations that leave the object unchanged when applied. Today, important topics in group theory are often illustrated to students using the Rubik's Cube.

84. Quaternion (1843)

Quaternions are four-dimensional numbers1 conceived in 1843 by Irish mathe -matician William Hamilton. Interestingly, Theodore Kaczynski (the "Unabomber") wrote intricate mathematical treatises on quaternion before he went on his killing spree. Quaternion can be represented in four dimensions by $Q = a_0 + a_1 i + a_2 j + a_3 k$ where i, j, and k are (like the imaginary number i) unit vectors in three orthogonal (perpendicular) directions, and they are perpendicular to the real number axis. To add or multiply two quaternion, we treat them as polynomials in *i,* j, and *k,* but use the following rules to deal with products $i^2 = j^2 = k^2 = -1$; $ij = -ji = k$; $jk = -kj = i$; and $ki = -ik = j$.



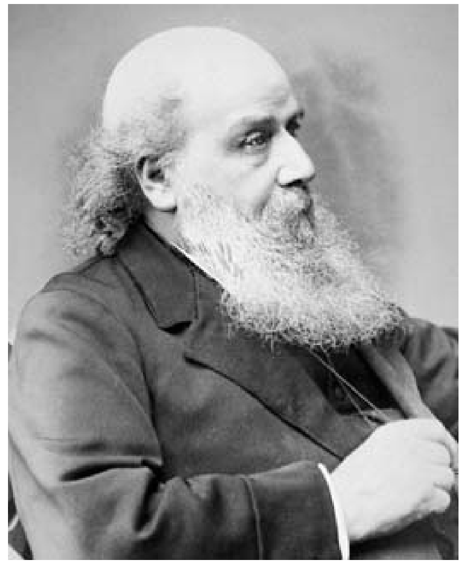
85. Catalan Conjecture (1844)

This history of the Catalan conjecture has a colorful cast of characters. Hundreds of years before Catalan, Frenchman Levi ben Gerson (1288-1 344) - better known as Gersonides or the Ralbag(a famous rabbi, philosopher, mathematician, and Talmudist) had already demonstrated a more restricted version of the conjecture, namely that the only powers of 2 and 3 that differ by 1 are 3^2 and 2^3. To set the stage for understanding the Catalan conjecture, consider the squares of whole numbers (integers) greater than 1, that is, 4, 9,16,25, ... and also consider the sequence of cubes, 8, 27, 64, 125 .... if we merge the two lists and place them in order, we obtain 4,8,9, 16,25,27, 36, ... Notice that 8 (the cube of 2) and 9 (the square of3) are consecutive integers. In 1844.

Belgian mathematician Eugene Catalan wrote a conjectured that 8 and 9 are the *only* powers of integers that are consecutive! If other such pairs had existed, they might have been found by searching for integer values for which $x^p - y^q = 1$ is true and for values of *x,* y, *p,* and *q* greater than 1. Catalan believed that only one solution exists: $3^2 - 2^3 = 1$.

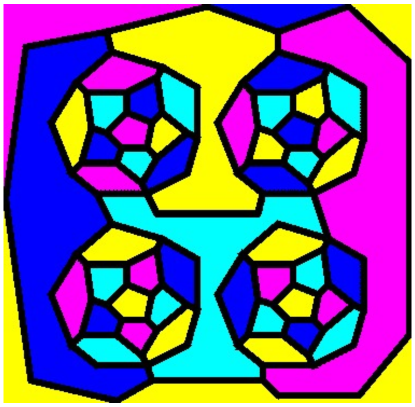
86. The Matrices of Sylvester (1850)

In 1850, in his paper "On a New Class of Theorems," British mathematician James Sylvester was the first to use the word *matrix* when referring to a rectangular arrangement, or array, of elements that can be added and multiplied. Matrices are often used to describe a system of linear equations or simply to represent information that depends on two or more parameters. Credit for understating and identifying the complete significance of the algebraic properties of matrices is given to the English mathematician Arthur Cayley for his later work on matrices in 1855. Because Cayley and Sylvester enjoyed many years of close collaboration, they are often considered the joint founders of matrix theory



Although matrix theory flourished in the mid-1800s, simple concepts of matrices date back to before the birth of Christ, when the Chinese knew of Magic Squares and also began to apply matrix methods to solve simultaneous equations. In the 1600s,
Japanese mathematician Seki Kowa (in 1683) and German mathematician Gottfried Leibnitz (in 1693) also explored the early use of matrices.
Cayley worked as a lawyer for more than a decade, while publishing about 250 Mathematics papers. During his time at Cambridge, he published another 650 papers. Cayley was first to introduce matrix multiplication.

87. Four-Color Theorem (1852)

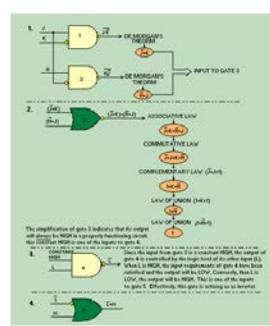


Mapmakers have believed for centuries that just four colors were sufficient for coloring any map drawn on a plane, so that no two distinct regions sharing a common edge are the same color, although two regions can share a common vertex and have the same color. Today, we know for certain that while some planar maps require fewer colors, no map requires more than four. Four colors are sufficient for maps drawn on spheres and cylinders. Seven colors are sufficient to paint any map on a torus (the surface of a doughnut shape).

In 1852, mathematician and botanist Francis Guthrie was the first to conjecture that four colors must be sufficient when he attempted to color a map of counties of England. Since the time of Guthne, mathematicians had tried in vain to *prove* the consequences of this seemingly simple four-color observation, and it remained one of the most famous unsolved problems in topology.
Finally, in 1976, mathematicians Kenneth Appel and Wolfgang Hakcn succeeded in proving the four-color theorem with the help of a computer testing thousands of cases, making it the first problem in pure mathematics to make use of a computer to produce an essential component for the proof.
Another is the classification of finite simple groups, embodied in a 1O,OOO-page multiauthor project. Alas, the traditional people-centered methods for ensuring that a proof is correct breaks down when a paper reaches thousands of pages.

88. Boolean algebra (1854)

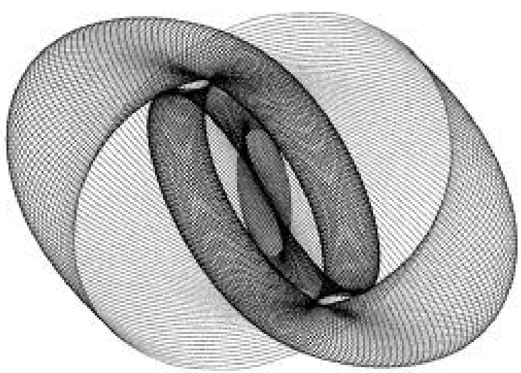
English mathematician George Boole's most important work was his 1854 *An Investigation into the Laws* of *Thought, on Which Are Founded the Mathematical Theories* of Logic *and Probabilities.* Boole was interested in reducing logic to a simple algebra involving just two quantities, 0 and I, and three basic operations: *and,* or, and *not.* A Boolean algebra (BA) is a set *A* together with binary operations + and · and a unary operation −, and elements 0, 1 of *A* such that the following laws hold: commutative and associative laws for addition and multiplication, distributive laws both for multiplication over addition and for addition over multiplication, and the following special laws:



x + (x · y) = x
x · (x + y) = x
x + (−x) = 1
x · (−x) = 0

Boole died at the age of 49 after he developed a bad fever.
Approximately seventy years after Boole's death, American mathematician Claude Shannon (1916-2001) was introduced to Boolean algebra, showed how Boolean algebra could be used to optimize the design of systems of telephone routing switches. He also demonstrated that circuits with relays could solve Boolean algebra problems.

89. Harmonograph (1857)

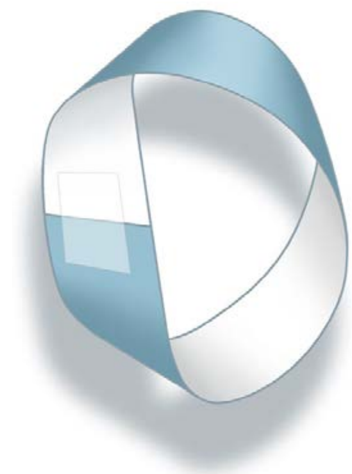


The first harmonographs were constructed in 1857, when French mathematician and physicist Jules Antoine Lissajous demonstrated patterns produced by two tuning forks, attached to small mirrors that vibrated at different frequencies. A beam of light reflected off the mirrors to produce the intricate curves that delighted a general public. British mathematician and physicist Hugh Blackburn is credited with making the first more traditional pendulum versions of the harmonograph, and many variations of Blackburn's hannonograph have been created up to the present day.

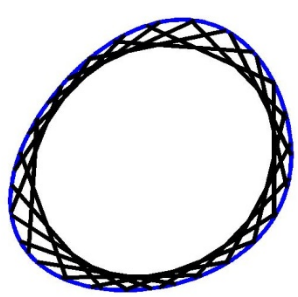
More complex harmonographs may employ additional pendulums that hang off one another. Harmonograph is a device consisting of two coupled pendula, usually oscillating at right angles to each other, which are attached to a pen. The resulting motion can produce beautiful, complicated curves which eventually terminate in a point as the motion of the pendula is damped by friction. In the absence of friction (and for small displacements so that the general pendulum equations of motion become simple harmonic motion

90. The Mobius Strip (1858)

German mathematician August Ferdinand Mobius was made the Mobius strip when he was almost seventy years old. Its impossible to color the mobius strip with different color at different sides of it. Years after Mobius's death, the popularity and applications of the strip grew, and it has become an integral part of mathematics, magic, science, art, engineering, literature, and music.
August Mobius had simultaneously discovered his famous strip with a contemporary scholar, the German mathematician Johann Benedict Listing (1808--1882). However, Mobius seems to have taken the concept a little further than Listing, as Mobius more closely explored some of the remarkable properties of this strip. The Mobius strip is the first one-sided surface discovered and investigated by humans. It seems far-fetched that no one had described the properties of one-sided surfaces until the mid-1800s, but history has recorded no such observations.



91. Holditch's Theorem (1858)



The theorem was published by Rev. Hamnet Holditch in 1858. Holditch was president of Caius College in Cambridge during the middle part of the 1800s. According to this theorem,

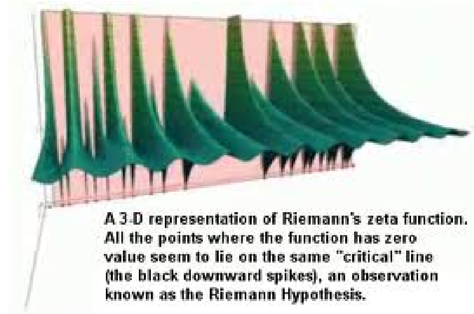
Let a chord of constant length be slid around a smooth, closed, convex curve C, and choose a point on the chord which divides it into segments of lengths p and q. This point will trace out a new closed curve C', as illustrated above. Provided certain conditions are met, the area between C and C' is given by $\pi p q$, as first shown by Holditch in 1858.

The Holditch curve for a circle of radius R is another circle which, from the theorem, has radius

$$r = \sqrt{R^2 - p q}\ .$$

92. Riemann Hypothesis (1859)

This conjecture is written by mathematician George Bernhard Riemann. First published in Riemann's groundbreaking 1859 paper (Riemann 1859),The distribution of such prime numbers among all natural numbers does not follow any regular pattern, however the German mathematician G.F.B. Riemann (1826 - 1866) observed that the frequency of prime numbers is very closely related to the behavior of an elaborate function



A 3-D representation of Riemann's zeta function. All the points where the function has zero value seem to lie on the same "critical" line (the black downward spikes), an observation known as the Riemann Hypothesis.

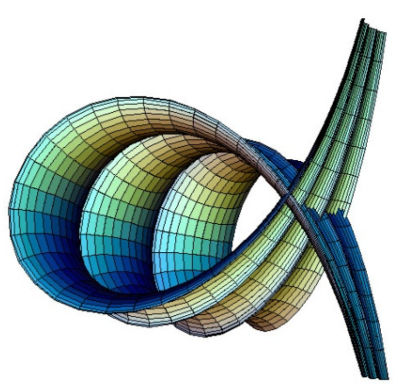
$\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + \ldots$

called the *Riemann Zeta function*. The Riemann hypothesis asserts that all *interesting* solutions of the equation

$\zeta(s) = 0$

lie on a certain vertical straight line. This has been checked for the first 1,500,000,000 solutions. A proof that it is true for every interesting solution would shed light on many of the mysteries surrounding the distribution of prime numbers.

93. Beltrami's Pseudosphere (1868)



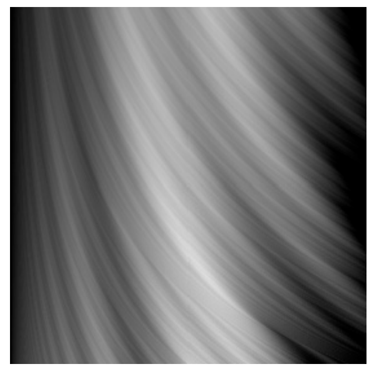
The pseudosphere is a geometrical object that resembles two musical horns glued together at their rims. The "mouthpieces" of the two horns are located at the ends of two inlinitely long tails, as if to be blown only by the omnipotent gods. The peculiar shape was lirst discussed in depth in the 1868 paper "Essay on an Interpretation of Non Euclidean Geometry" by Italian mathematician Eugenio Beltrami, famous for his work in geometry and physics. To produce the surface, a curve called a *tractrix* is rotated about its asymptote. Whereas an ordinary sphere has a property called *poSitive curvature* everywhere on its surface, a pseudosphere has a constant negative curvature, which means that it can be thought of as maintaining a constant concavity over its entire surface (except at its central cusp).

Thus, a sphere is a closed surface with a finite area, while a

pseudosphere is an open surface with infinite area. The negative curvature of a pseudosphere requires that the angles of a triangle drawn on its surface add up to less than 180°. The geometry of the pseudosphere is called *hyperbolic,* and some astronomers in the past have suggested that our entire universe might be described by hyperbolic geometry with properties of a pseudosphere. The pseudosphere is of historical importance because it was one of the lirst models for a Non-Euclidean space.

94. Weierstrass Function (1872)

In the early 1800s, mathematicians often thought of a continuous function *f(x)* as having a derivative (a unique tangent line) that could be specified along most points in the curve. In 1872, German mathematician Karl Weierstrass stunned mathematical colleagues at the Berlin Academy by proving this thinking to be false.



His function, which was continuous everywhere but differentiable (possessing a derivative) nowhere, was defined by $f(x) = \sum a^k \cos(b^k \pi x)$, where the sum is from $k = 0$ to ∞. Here, a is a real number with $0 < a < 1$, b is an odd positive integer, and $ab > (1+3\pi/2)$.. The summation symbol indicates that the function is constructed from an infinite number of trigonometric functions to produce a densely nested oscillating structure. Mathematicians were well aware that functions might not be differentiable at a few troublesome points, such as the bottom of the inverted wedge shape specified by $f(x) = |x|$ which has no derivative at x = O. However, after Weierstrass's demonstration of a nowhere-differentiable curve, mathematicians were in a quandary.

In 1875, Paul du Bois-Reymond published the Weierstrass function, making it the first published function of its kind. Other mathematicians, such as Czech mathematician Bernard Bolzano and German mathematician Bernhard Riemann, had worked on similar (unpublished) constructions in 1830 and 1861, respectively. Another example of an everywhere-continuous but nowhere-differentiable curve is the fractal Koch curve.

95. Fifteen Puzzle (1874)

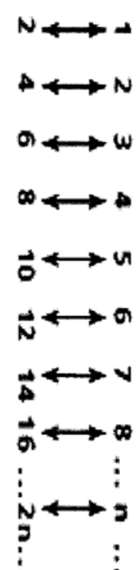
A sliding tile puzzle invented by Sam Loyd that became worldwide obsession. Fifteen little tiles ,numbered 1 to 15,were placed in 4×4 frame in serial order except for tile 14 & 15;which were swapped around; the lower right hand square was left empty. The object of puzzle was to get all the tiles in correct order; the only allowed moves were sliding counters into the empty square.

The game even made its way into the solemn House halls of German Parliament.

96.Cantor's Transfinite Number (1874)

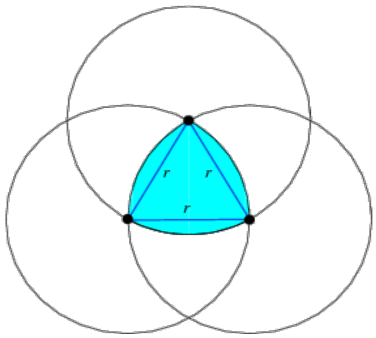
Georg Cantor was born in St. Petersburg in 1845 but lived in Germany for most of his life, teaching at the University of Halle. He spent much of his life dealing with the science of infinite-both the infinitely small and infinitely large.

Thinking infinity poses a great challenge for the most of us, because we are accustomed to thinking only about finite sets. Cantor realized that properties that apply to infinite sets may differ greatly from those that apply to finite numbers. He discovered that the "infinite class" is characterized by the property that the Whole may not be greater than the any of its parts. e.g.:- He showed that total no. of integers, even and odd, is same as the no. of even integers, using a pairing like as shown in figure.



He described such infinite sets of no. as "Countable," or denumerably infinite.He assigned a no. to represent the cardinality of set of all integers. This no. was the first so called transfinite number, used to denote the cardinality of countable infinite sets of numbers.

97. Reuleaux Triangle (1875)



A Reuleaux Triangle is the shape enclosed by three 60® arc around an equilateral triangle, where each arc is drawn radically from one of the vertices. This region has a constant width equal to the side of triangle and so can roll along a road or inside a square. The square Hole code in the electronic supplement generates an animation of Reuleaux polygon inside a square. The Reuleaux triangle has the smallest area for a given width of any curve of constant width. Quote from Paul Anderson's Fiction –"Three Cornered Wheel .

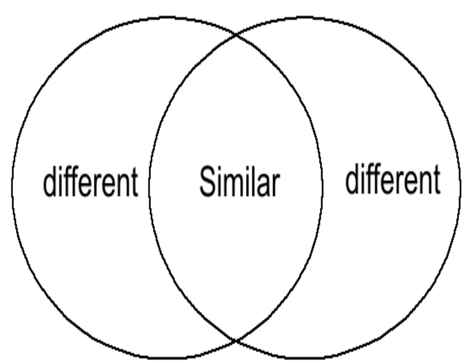
"Draw an equilateral triangle, ABC. Put the point of your compasses on A and draw the arc BC. Move to B and describe AC, then to C and describe AB. Round off the corners. The resulting figure has constant width. It will roll between two parallel lines tangent to it maintaining that tangency for the whole revolution. As a matter of fact, the class of constant-width polygons is infinite. The circle is merely a limiting case."

The term derives from Franz Reuleaux, a 19th-century German engineer who did pioneering work on ways that machines translate one type of motion into another, although the concept was known before his time.

98. Venn Diagram (1880)

In 1880 John Venn(1834–1923) introduced Venn diagram in a paper entitled *On the Diagrammatic and Mechanical Representation of Propositions and Reasoning's* in the "Philosophical Magazine and Journal of Science", about the different ways to represent propositions by diagrams.

Venn himself did not use the term "Venn diagram" and referred to his invention as "Eulerian Circles."

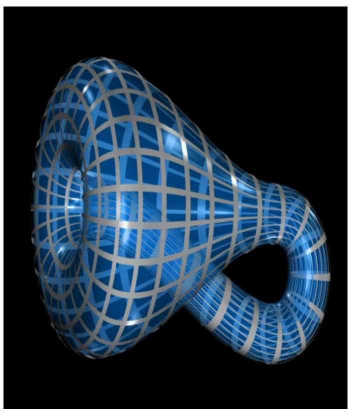


For example, in the opening sentence of his 1880 article Venn writes, "Schemes of diagrammatic representation have been so familiarly introduced into logical treatises during the last century or so, that many readers, even those who have made no professional study of logic, may be supposed to be acquainted with the general nature and object of such devices.

The first to use the term "Venn diagram" was Clarence Irving Lewis in 1918, in his book "A Survey of Symbolic Logic".

Venn diagrams are very similar to Euler Diagrams, which were invented by Leonhard Euler (1708–1783) in the 18th century. M. E. Baron has noted that Leibniz (1646–1716) in the 17th century produced similar diagrams before Euler, but much of it was unpublished. She also observes even earlier Euler-like diagrams by Ramon Lull in the 13th Century.

99. Klein Bottle (1882)



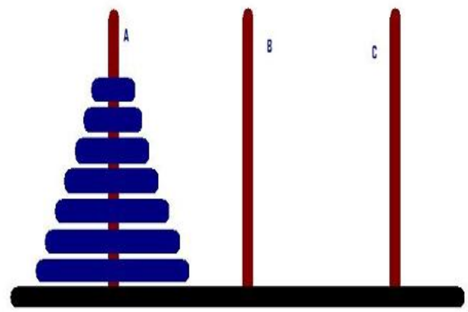
The Klein bottle is an object in which the flexible neck of a bottle wraps into the bottle to form a shape with no inside an outside. Further, it is an example of a non-oriented surface or in two dimensional manifolds the notion of left and right cannot be consistently defined.

The Klein bottle was first described in 1882 by the German mathematician Felix Klein(1849-1925).Because of the peculiar properties of the klein bottle, mathematicians and puzzle enthusiasts study chess games and mazes played on klein bottle surfaces. If a map were drawn on a klein bottle, six different colors would be needed to ensure that no bordering areas would be colored the same.

100. Tower Of Hanoi (1883)

The Tower of Hanoi was invented by French Mathematician Edouard Lucas in 1883 and sold as a toy. This mathematical puzzle consists of several disks of different sizes that slide onto any of three pegs.

The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:



1. Only one disk may be moved at a time.
2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack.
3. No disk may be placed on top of a smaller disk.

With three disks, the puzzle can be solved in seven moves. The minimum number of moves required to solve a Tower of Hanoi puzzle is $2^n - 1$, where *n* is the number of disks.

There is a mythical story about an Indian temple in Kashi Vishwanath which contains a large room with three time-worn posts in it surrounded by 64 golden disks. Brahmin priests, acting out the command of an ancient prophecy, have been moving these disks, in accordance with the immutable rules of the Brahma, since that time. The puzzle is therefore also known as the Tower of Brahma puzzle. According to the legend, when the last move of the puzzle will be completed, the world will end.

If the legend were true, and if the priests were able to move disks at a rate of one per second, using the smallest number of moves, it would take them $2^{64}-1$ seconds or roughly 585 billion year or 18,446,744,073,709,551,615 turns to finish, or about 127 times the current age of the sun.

There are many variations on this legend. For instance, in some telling, the temple is a monastery and the priests are monks. The temple or monastery may be said to be in different parts of the world — including Hanoi, Vietnam, and may be associated with any religion. In some versions, other elements are introduced, such as the fact that the tower was created at the beginning of the world, or that the priests or monks may make only one move per day.

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